EXERCISE

SHORT ANSWER TYPE QUESTIONS

Q1. The first term of an A.P. is a, and the sum of the first p terms is zero, show that the sum of its next q terms is $\dfrac{-a(p+q)q}{p-1}$

Sol. Given that $a_1 = a$ and $S_p = 0$

Sum of next q terms of the given A.P. $= S_{p+q} - S_p$

$$\therefore \quad S_{p+q} = \frac{p+q}{2}[2a + (p+q-1)d]$$

$$\text{and} \quad S_p = \frac{p}{2}[2a + (p-1)d] = 0$$

$$\Rightarrow \quad 2a + (p-1)d = 0 \quad \Rightarrow \quad (p-1)d = -2a$$

$$\Rightarrow \quad d = \frac{-2a}{p-1}$$

Sum of next q terms $= S_{p+q} - S_p$

$$= \frac{p+q}{2}[2a + (p+q-1)d] - 0$$

$$= \frac{p+q}{2}\left[2a + (p+q-1)\left(\frac{-2a}{p-1}\right)\right]$$

$$= \frac{p+q}{2}\left[2a + \frac{(p-1)(-2a)}{p-1} - \frac{2aq}{p-1}\right]$$

$$= \frac{p+q}{2}\left[2a - 2a - \frac{2aq}{p-1}\right] = \frac{(p+q)}{2}\left(\frac{-2aq}{p-1}\right)$$

$$= \frac{-a(p+q)q}{p-1}$$

Hence, the required sum $= \dfrac{-a(p+q)q}{p-1}$

Q2. A man saved ₹ 66000 in 20 years. In each succeeding year after the first year, he saved ₹ 200 more than what he saved in the previous year. How much did he save in the first year?

Sol. Let ₹ x be saved in first year.

Annual increment = ₹ 200

which forms an A.P.

first term = a and common difference $d = 200$

$n = 20$ years

$$\therefore \quad S_n = \frac{n}{2}[2a + (n-1)d] \quad \Rightarrow \quad S_{20} = \frac{20}{2}[2a + (20-1)200]$$

$$\Rightarrow \quad 66000 = 10[2a + 3800] \quad \Rightarrow \quad 6600 = 2a + 3800$$

$$\Rightarrow \quad 2a = 6600 - 3800 \quad \Rightarrow \quad 2a = 2800 \quad \Rightarrow \quad a = 1400$$

Hence, the man saved ₹ 1400 in the first year.

Q3. A man accepts a position with an initial salary of ₹ 5200 per month. It is understood that he will receive an automatic increase of ₹ 320 in the every next month and each month thereafter.

(a) Find his salary for the tenth month;

(b) What is his total earnings during the first year?

Sol. Given that fixed increment in the salary of a man

= ₹ 320 each month

Initial salary = ₹ 5200 which makes an A.P.

whose first term (a) = ₹ 5200 and common difference (d) = ₹ 320

(i) Salary for the tenth month

$$a_{10} = a + (n-1)d$$
$$= 5200 + (10-1) \times 320 = 5200 + 2880 = ₹\ 8080$$

(ii) Total earning during the first year (12 months)

$$S_{12} = \frac{12}{2}[2 \times 5200 + (12-1) \times 320]$$

$$\left[\because \quad S_n = \frac{n}{2}[2a + (n-1)d]\right]$$

$$= 6[10400 + 3520] = 6 \times 13920 = ₹\ 83520$$

Hence, the required amount is (i) ₹ 8080 (ii) ₹ 83520.

Q4. If the pth and qth terms of a G.P. are q and p respectively, then show that its $(p+q)$th term is $\left(\frac{q^p}{p^q}\right)^{\frac{1}{p-q}}$

Sol. Let a be the first term and r be the common ratio of a G.P.

Given that $a_p = q \Rightarrow ar^{p-1} = q$ ...(i)

and $a_q = p \Rightarrow ar^{q-1} = p$ ...(ii)

Dividing eq. (i) by eq. (ii) we get,

$$\frac{ar^{p-1}}{ar^{q-1}} = \frac{q}{p} \quad \Rightarrow \quad \frac{r^{p-1}}{r^{q-1}} = \frac{q}{p}$$

$$\Rightarrow \quad r^{p-q} = \frac{q}{p} \quad \Rightarrow \quad r = \left(\frac{q}{p}\right)^{\frac{1}{p-q}}$$

putting the value of r in eq. (i), we get

$$a\left[\frac{q}{p}\right]^{\frac{1}{p-q}\times p-1} = q$$

$$a\left[\frac{q}{p}\right]^{\frac{p-1}{p-q}} = q$$

$$\therefore \qquad a = q\cdot\left[\frac{p}{q}\right]^{\frac{p-1}{p-q}}$$

Now $\quad T_{p+q} = ar^{p+q-1} = q\left[\frac{p}{q}\right]^{\frac{p-1}{p-q}}\left[\frac{q}{p}\right]^{\frac{1}{p-q}(p+q-1)}$

$$= q\left(\frac{p}{q}\right)^{\frac{p-1}{p-q}}\cdot\left(\frac{q}{p}\right)^{\frac{p+q-1}{p-q}} = q\left(\frac{p}{q}\right)^{\frac{p-1}{p-q}}\cdot\left(\frac{p}{q}\right)^{\frac{-(p+q-1)}{p-q}}$$

$$= q\left(\frac{p}{q}\right)^{\frac{p-1}{p-q}-\frac{p+q-1}{p-q}} = q\left(\frac{p}{q}\right)^{\frac{p-1-p-q+1}{p-q}}$$

$$= q\left(\frac{p}{q}\right)^{\frac{-q}{p-q}} = q\left(\frac{q}{p}\right)^{\frac{q}{p-q}} = \frac{q^{\frac{q}{p-q}+1}}{p^{\frac{q}{p-q}}}$$

$$= \frac{q^{\frac{p}{p-q}}}{p^{\frac{q}{p-q}}} = \left[\frac{q^p}{p^q}\right]^{\frac{1}{p-q}}$$

Hence, the required term = $\left[\frac{q^p}{p^q}\right]^{\frac{1}{p-q}}$.

Q5. A Carpenter was hired to build 192 window frames. The first day, he made five frames and each day, thereafter he made two more frames than he made the day before. How many days did it take him to finish the job?

Sol. Here, first term $a = 5$ and the common difference $d = 2$ let the carpenter will take n days to finish the job

$$S_n = 192$$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$192 = \frac{n}{2}[2\times 5+(n-1)2]$$

$$\Rightarrow \quad 192\times 2 = n[10+2n-2] \quad\Rightarrow\quad 384 = n(2n+8)$$

$$\Rightarrow \quad 384 = 2n^2+8n \quad\Rightarrow\quad 2n^2+8n-384 = 0$$

$\Rightarrow \quad n^2 + 4n - 192 = 0 \quad \Rightarrow \quad n^2 + 16n - 12n - 192 = 0$

$\Rightarrow n(n + 16) - 12\,(n + 16) = 0 \quad \Rightarrow \quad (n - 12)\,(n + 16) = 0$

$\Rightarrow \quad n = 12 \qquad [\because \quad n \neq -16]$

Hence, the required number of days = 12.

Q6. We know the sum of the interior angles of a triangle is 180°. Show that the sums of the interior angles of a polygons with 3, 4, 5, 6, … sides form an arithmetic progression. Find the sum of the interior angles for a 21 sides polygon.

Sol. Since, the sum of all interior angles of a polygon of n sides

$= (2n - 4) \times 90°$

$\therefore$ Sum of interior angles of a polygon of sides 3

$= (2 \times 3 - 4) \times 90° = 180°$

Sum of interior angles of a polygon of sides 4

$= (2 \times 4 - 4) \times 90° = 360°$

Similarly, the sum of interior angles of the polygon of sides, 5, 6, 7 … are 540°, 720°, 900°…

Therefore, the series will be 180°, 360°, 540°, 720°, 900°… which is A.P.

Here $a = 180°$, $d = 180°$

We have to find the sum of interior angles of a polygon of 21 sides *i.e.* 19th term

$a_n = a + (n - 1)d$

$a_{19} = 180° + (19 - 1)180° = 180° + 18 \times 180°$

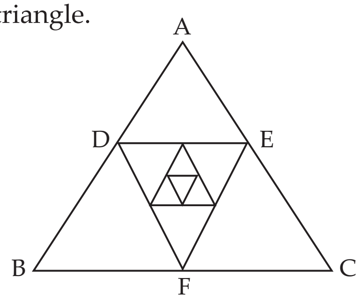
$= 180° + 3240° = 3420°$

Hence, the required sum of interior angles = 3420°.

Q7. A side of an equilateral triangle is 20 cm long. A second equilateral triangle is inscribed in it by joining the mid points of the sides of the first triangle. The process is continued as shown in the accompanying diagram. Find the perimeter of the sixth inscribed equilateral triangle.

Sol. The side of the first equilateral $\Delta ABC = 20$ cm

By joining the mid points of the sides of this triangle, we get the second equilateral triangle which each side $= \dfrac{20}{2} = 10$ cm



$[\because$ The line joining the mid-points of two sides of a triangle is 1/2 and parallel to third side of the triangle$]$

Similarly each side of the third equilateral triangle = $\frac{10}{2} = 5$ cm

$\therefore$ Perimeter of first triangle = $20 \times 3 = 60$ cm

Perimeter of the second triangle = $10 \times 3 = 30$ cm

and the perimeter of the third triangle = $5 \times 3 = 15$ cm

Therefore, the series will be 60, 30, 15, …

which is G.P. in which $a = 60$, and $r = \frac{30}{60} = \frac{1}{2}$

Now, we have to find the perimeter of the sixth inscribed equilateral triangle

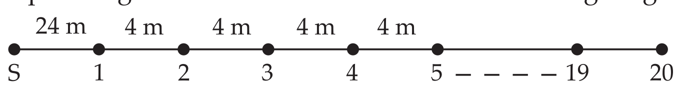
$$\therefore \quad a_6 = ar^{6-1}$$

$$= 60 \times \left(\frac{1}{2}\right)^5 = 60 \times \frac{1}{32} = \frac{15}{8} \text{ cm}$$

Hence, the required perimeter = $\frac{15}{8}$ cm

Q8. In a potato race 20 potatoes are placed in a line at intervals of 4 metres with the first potato 24 metres from the starting point. A contestant is required to bring the potatoes back to the starting place one at a time. How far would he runs in bringing back all the potatoes?

Sol. As per the given information we have the following diagram



Starting point = S

Distance travelled to bring the first potato = 24 + 24 = 48 m

Distance travelled to bring the second potato = 2(24 + 4) = 56 m

Distance travelled to bring the third potato = 2(24 + 4 + 4) = 64 m

Therefore, the series will be = 48, 56, 64, …

which an A.P. in which $a = 48$, $d = 56 - 48 = 8$

We have to find the total distance to bring all the potatoes back, so, $n = 20$

$$\therefore \quad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow \quad S_{20} = \frac{20}{2}[2 \times 48 + (20-1)8] = 10[96 + 152]$$

$$= 10 \times 248 = 2480 \text{ m}$$

Hence, the required distance = 2480 m

Q9. In a cricket tournament 16 school teams participated. A sum of ₹ 8000 is to be awarded among themselves as prize money. If the last placed team is awarded ₹ 275 in prize money and the award increases by the same amount for successive finishing place, how much amount will the first place team receive?

Sol. Let the prize amount got by first place team be ₹ a

Since, the prize money increases by the same amount for successive finishing places, therefore the series will be A.P.

$\therefore \quad a_n = 275,\ n = 16$ and $S_{16} = 8000$

$$a_n = a + (n-1)d$$

$$275 = a + (16-1)(-d)$$

[$\because$ Common difference d is (–) as the series is decreasing]

$\Rightarrow \quad 275 = a - 15d \qquad ...(i)$

Now $\quad S_n = \dfrac{n}{2}[2a + (n-1)d]$

$$S_{16} = \frac{16}{2}[2a + 15(-d)]$$

$\Rightarrow \quad 8000 = 8[2a - 15d] \quad \Rightarrow \quad 2a - 15d = 1000 \qquad ...(ii)$

Solving eq. (i) and eq. (ii) we get

$$a = 725 \quad \text{and } d = 30$$

Hence, the required award received by first place term = ₹ 725.

Q10. If $a_1, a_2, a_3, \ldots, a_n$ are in A.P. where $a_i > 0$ for all *i.e.* show that

$$\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \cdots \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} = \frac{n-1}{\sqrt{a_1} + \sqrt{a_n}}$$

Sol. Given that $a_1, a_2, a_3, a_4, \ldots, a_n$ are in A.P.

$\therefore$ Common difference $d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots = a_n - a_{n-1}$

If $a_2 - a_1 = d$ then $\sqrt{a_2^2} - \sqrt{a_1^2} = d$

$\Rightarrow \quad \left(\sqrt{a_2} - \sqrt{a_1}\right)\left(\sqrt{a_2} + \sqrt{a_1}\right) = d \quad [\because \quad a^2 - b^2 = (a+b)(a-b)]$

$\Rightarrow \quad \dfrac{1}{\sqrt{a_1} + \sqrt{a_2}} = \dfrac{\sqrt{a_2} - \sqrt{a_1}}{d}$

Similarly $\quad \dfrac{1}{\sqrt{a_2} + \sqrt{a_3}} = \dfrac{\sqrt{a_3} - \sqrt{a_2}}{d}$

$$\frac{1}{\sqrt{a_3} + \sqrt{a_4}} = \frac{\sqrt{a_4} - \sqrt{a_3}}{d}$$

$$\cdots \quad \cdots \quad \cdots$$

$$\frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} = \frac{\sqrt{a_n} - \sqrt{a_{n-1}}}{d}$$

Adding the above terms, we get

$$\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \frac{1}{\sqrt{a_3} + \sqrt{a_4}} + \cdots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}}$$

$$= \frac{1}{d}[\sqrt{a_2} - \sqrt{a_1} + \sqrt{a_3} - \sqrt{a_2} + \sqrt{a_4} - \sqrt{a_3} + \cdots \sqrt{a_n} - \sqrt{a_{n-1}}]$$

$$= \frac{1}{d}[\sqrt{a_n} - \sqrt{a_1}] \quad \text{...}(i)$$

Now $\quad a_n = a_1 + (n-1)d$

$\Rightarrow \quad a_n - a_1 = (n-1)d$

$\Rightarrow \quad \sqrt{a_n^2} - \sqrt{a_1^2} = (n-1)d$

$\Rightarrow \quad (\sqrt{a_n} + \sqrt{a_1})(\sqrt{a_n} - \sqrt{a_1}) = (n-1)d$

$\Rightarrow \quad \sqrt{a_n} - \sqrt{a_1} = \dfrac{(n-1)d}{\sqrt{a_n} + \sqrt{a_1}}$

$\Rightarrow \quad \dfrac{\sqrt{a_n} - \sqrt{a_1}}{d} = \dfrac{n-1}{\sqrt{a_n} + \sqrt{a_1}} \quad \text{...}(ii)$

From Eq. (i) and eq. (ii) we get

$$\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \frac{1}{\sqrt{a_3} + \sqrt{a_4}} + \cdots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} = \frac{n-1}{\sqrt{a_n} + \sqrt{a_1}}$$

Hence proved.

Q11. Find the sum of the series

$(3^3 - 2^3) + (5^3 - 4^3) + (7^3 - 6^3) + \cdots$ to (i) n terms (ii) 10 terms

Sol. Given series

$\Rightarrow \quad (3^3 - 2^3) + (5^3 - 4^3) + (7^3 - 6^3) + \cdots$

$= (3^3 + 5^3 + 7^3 + \cdots) - (2^3 + 4^3 + 6^3 + \cdots)$

$\Rightarrow \quad [3^3 + 5^3 + 7^3 + \ldots (2n+1)^3] - [2^3 + 4^3 + 6^3 + \cdots (2n)^3]$

$\therefore \quad T_n = (2n+1)^3 - (2n)^3$

$= (2n + 1 - 2n)[(2n+1)^2 + (2n+1)(2n) + (2n)^2]$

$[\because \quad a^3 - b^3 = (a-b)(a^2 + ab + b^2)]$

$= 1 \cdot [4n^2 + 1 + 4n + 4n^2 + 2n + 4n^2]$

$= 12n^2 + 6n + 1$

(i) $\quad S_n = \sum T_n = 12\sum n^2 + 6\sum n + n$

$= 12 \cdot \dfrac{n(n+1)(2n+1)}{6} + \dfrac{6n(n+1)}{2} + n$

$= 2n(n+1)(2n+1) + 3n(n+1) + n$

$= n[2(n+1)(2n+1) + 3(n+1) + 1]$

$= n[2(2n^2 + 3n + 1) + 3n + 3 + 1]$

$= n[4n^2 + 6n + 2 + 3n + 4] = n[4n^2 + 9n + 6]$

$= 4n^3 + 9n^2 + 6n$

(*ii*) $S_{10} = 4(10)^3 + 9(10)^2 + 6(10) = 4 \times 1000 + 900 + 60$
$= 4000 + 960 = 4960$

Q12. Find the rth term of an A.P. sum of whose first n terms is $2n + 3n^2$

Sol. Given that $S_n = 2n + 3n^2$

$$\Rightarrow \quad S_1 = 2 \times 1 + 3(1)^2 = 5$$
$$\Rightarrow \quad S_2 = 2 \times 2 + 3 \times 4 = 16$$
$$\Rightarrow \quad S_3 = 2 \times 3 + 3 \times 9 = 33$$
$$\cdots \quad \cdots \quad \cdots$$
$$\therefore \quad S_1 = a_1 = 5$$
$$S_2 - S_1 = a_2 = 16 - 5 = 11$$
$$\therefore \quad d = a_2 - a_1 = 11 - 5 = 6$$

Now $T_r = a_1 + (r-1)d$
$= 5 + (r-1)6 = 5 + 6r - 6 = 6r - 1$

Hence, the required rth term is $6r - 1$

LONG ANSWER TYPE QUESTIONS

Q13. If A is the arithmetic mean and G_1, G_2 be two geometric means between any two numbers, then prove that $2A = \frac{G_1^2}{G_2} + \frac{G_2^2}{G_1}$.

Sol. Let the two numbers be x and y

$$\therefore \quad A = \frac{x+y}{2} \quad \ldots(i)$$

If G_1 and G_2 be the geometric means between x and y then x, G_1, G_2, y are in G.P.

then $y = xr^{4-1}$ $\quad [\because \quad a_n = ar^{n-1}]$

$$\Rightarrow \quad y = xr^3 \quad \Rightarrow \quad \frac{y}{x} = r^3$$

$$\Rightarrow \quad r = \left(\frac{y}{x}\right)^{1/3}$$

Now $G_1 = xr = x\left(\frac{y}{x}\right)^{1/3}$ $\quad \left[\because \quad r = \left(\frac{y}{x}\right)^{1/3}\right]$

and $G_2 = xr^2 = x\left(\frac{y}{x}\right)^{2/3}$

$$\therefore \text{ from RHS} \quad \frac{G_1^2}{G_2} + \frac{G_2^2}{G_1} = \frac{x^2\left(\frac{y}{x}\right)^{2/3}}{x\left(\frac{y}{x}\right)^{2/3}} + \frac{x^2\left(\frac{y}{x}\right)^{4/3}}{x\left(\frac{y}{x}\right)^{1/3}}$$

$$= x + x\left(\frac{y}{x}\right)^{\frac{4}{3}-\frac{1}{3}} = x + x\left(\frac{y}{x}\right)$$

$= x + y = 2A$ LHS. [using eq. (i)]

$\therefore$ LHS = RHS Hence proved.

Q14. If $\theta_1, \theta_2, \theta_3, \ldots, \theta_n$ are in A.P., whose common difference is d, show that

$$\sec\theta_1 \cdot \sec\theta_2 + \sec\theta_2 \sec\theta_3 + \cdots + \sec\theta_{n-1} \cdot \sec\theta_n = \frac{\tan\theta_n - \tan\theta_1}{\sin d}$$

Sol. Since $\theta_1, \theta_2, \theta_3, \ldots, \theta_n$ are in A.P.

$\therefore$ $\theta_2 - \theta_1 = \theta_3 - \theta_2 = \ldots = \theta_n - \theta_{n-1} = d$

Now we have to prove that

$$\sec\theta_1 \cdot \sec\theta_2 + \sec\theta_2 \cdot \sec\theta_3 + \cdots + \sec\theta_{n-1} . \sec\theta_n = \frac{\tan\theta_n - \tan\theta_1}{\sin d} \quad \text{LHS.}$$

$$\Rightarrow \frac{\sin d}{\sin d}[\sec\theta_1 \cdot \sec\theta_2 + \sec\theta_2 \sec\theta_3 + \cdots + \sec\theta_{n-1} \cdot \sec\theta_n]$$

Taking only $\dfrac{\sin d[\sec\theta_1 \cdot \sec\theta_2]}{\sin d} = \dfrac{\sin d\left[\dfrac{1}{\cos\theta_1} \cdot \dfrac{1}{\cos\theta_2}\right]}{\sin d}$

$$= \frac{\sin(\theta_2 - \theta_1)}{\sin d} \cdot \frac{1}{\cos\theta_1 \cos\theta_2}$$

$$= \frac{1}{\sin d}\left[\frac{\sin\theta_2 \cos\theta_1 - \cos\theta_2 \sin\theta_1}{\cos\theta_1 \cos\theta_2}\right]$$

$$= \frac{1}{\sin d}\left[\frac{\sin\theta_2 \cos\theta_1}{\cos\theta_1 \cos\theta_2} - \frac{\cos\theta_2 \sin\theta_1}{\cos\theta_1 \cos\theta_2}\right]$$

$$= \frac{1}{\sin d}[\tan\theta_2 - \tan\theta_1]$$

Similarly we can solve other terms which will be

$$\frac{1}{\sin d}[\tan\theta_3 - \tan\theta_2] \text{ and } \frac{1}{\sin d}[\tan\theta_4 - \tan\theta_3]$$

Here LHS $= \dfrac{1}{\sin d}[\tan\theta_2 - \tan\theta_1 + \tan\theta_3 - \tan\theta_2 + \cdots + \tan\theta_n - \tan\theta_{n-1}]$

$$= \frac{1}{\sin d}[-\tan\theta_1 + \tan\theta_n] = \frac{\tan\theta_n - \tan\theta_1}{\sin d} \quad \text{RHS.}$$

LHS = RHS Hence proved.

Q15. If the sum of p terms of an A.P. is q and q terms is p, show that the sum of $p + q$ terms is $-(p + q)$.

Sol. Let a be the first term and d the common difference of the given A.P.

$\therefore \quad S_p = \frac{p}{2}[2a + (p-1)d] = q \quad \Rightarrow \quad 2a + (p-1)d = \frac{2q}{p} \quad ...(i)$

and $\quad S_q = \frac{q}{2}[2a + (q-1)d] = p \quad \Rightarrow \quad 2a + (q-1)d = \frac{2p}{q} \quad ...(ii)$

Subtracting eq. (*ii*) from eq. (*i*) we get

$$(p-q)d = \frac{2q}{p} - \frac{2p}{q} \Rightarrow (p-q)d = \frac{2(q^2 - p^2)}{pq}$$

$$\Rightarrow \quad (p-q)d = \frac{-2}{pq}(p^2 - q^2)$$

$$\Rightarrow \quad (p-q)d = \frac{-2}{pq}(p+q)(p-q) \quad \Rightarrow \quad d = \frac{-2}{pq}(p+q)$$

Substituting the value of d in eq. (*i*) we get

$$2a + (p-1)\left[\frac{-2(p+q)}{pq}\right] = \frac{2q}{p}$$

$$\Rightarrow \quad 2a = \frac{2q}{p} + \frac{2(p-1)(p+q)}{pq} \quad \Rightarrow \quad a = \frac{q}{p} + \frac{(p-1)(p+q)}{pq}$$

$$\Rightarrow \quad a = \frac{q^2 + p^2 + pq - p - q}{pq}$$

Now $\quad S_{p+q} = \frac{p+q}{2}[2a + (p+q-1)d]$

$$= \frac{p+q}{2}\left[\frac{2q^2 + 2p^2 + 2pq - 2p - 2q}{pq} + \frac{(p+q-1)[-2(p+q)]}{pq}\right]$$

$$= \frac{p+q}{2}\left[\frac{2q^2 + 2p^2 + 2pq - 2p - 2q - 2p^2 - 2pq + 2p - 2pq - 2q^2 + 2q}{pq}\right]$$

$$= \frac{p+q}{2}\left[\frac{-2pq}{pq}\right] = -(p+q) \quad \text{Hence proved.}$$

Q16. If pth, qth and rth terms of an A.P. and G.P. are both a, b, and c respectively. Show that

$$a^{b-c} \cdot b^{c-a} \cdot c^{a-b} = 1$$

Sol. Let A and d be the first term and common difference respectively of an A.P. and x and R be the first term and common ratio respectively of the G.P.

$\therefore \quad A + (p-1)d = a \quad ...(i)$

$A + (q-1)d = b \quad ...(ii)$

and $\quad A + (r-1)d = c \quad ...(iii)$

For G.P., we have

$$xR^{p-1} = a \quad ...(iv)$$
$$xR^{q-1} = b \quad ...(v)$$
and $$xR^{r-1} = c \quad ...(vi)$$

Subtracting eq. (*ii*) from eq. (*i*) we get

$$(p-q)d = a-b \quad ...(vii)$$

Similarly, $$(q-r)d = b-c \quad ...(viii)$$

and $$(r-p)d = c-a \quad ...(ix)$$

Now we have to prove that

$$a^{b-c} \cdot b^{c-a} \cdot c^{a-b} = 1$$

L.H.S. $a^{b-c} \cdot b^{c-a} \cdot c^{a-b}$

$$= [xR^{p-1}]^{(q-r)d} \cdot [xR^{q-1}]^{(r-p)d} \cdot [xR^{r-1}]^{(p-q)d}$$

[from (*i*), (*ii*), (*iii*), (*iv*), (*v*), (*vi*), (*vii*), (*viii*), (*ix*)]

$$= x^{(q-r)d} \cdot R^{(p-1)\,(q-r)d} \cdot x^{(r-p)d} \cdot R^{(q-1)\,(r-p)d} \cdot x^{(p-q)d} \cdot R^{(r-1)\,(p-q)d}$$
$$= x^{(q-r)d + (r-p)d + (p-q)d}\, R^{(p-1)(q-r)d + (q-1)\,(r-p)d + (r-1)\,(p-q)d}$$
$$= x^{(q-r+r-p+p-q)d} \cdot R^{(pq-pr-q+r+qr-pq-r+p+pr-qr-p+q)d}$$
$$= x^{(0)d} \cdot R^{(0)d} = x^0 \cdot R^0 = 1 \quad \text{R.H.S.}$$

L.H.S. = R.H.S. Hence proved.

OBJECTIVE TYPE QUESTIONS

Q17. If the sum of n terms of an A.P. is given by $S_n = 3n + 2n^2$, then the common difference of the A.P. is

(*a*) 3 (*b*) 2 (*c*) 6 (*d*) 4

Sol. Given that $S_n = 3n + 2n^2$

$$S_1 = 3(1) + 2(1)^2 = 5$$
$$S_2 = 3(2) + 2(4) = 14$$
$$S_1 = a_1 = 5$$
$$S_2 - S_1 = a_2 = 14 - 5 = 9$$

$\therefore$ Common difference $d = a_2 - a_1 = 9 - 5 = 4$

Hence, the correct option is (*d*).

Q18. The third term of G.P. is 4. The product of its first 5 terms is

(*a*) 4^3 (*b*) 4^4
(*c*) 4^5 (*d*) None of these

Sol. Given that $T_3 = 4$

$\Rightarrow$ $ar^{3-1} = 4$ $[\because \quad T_n = ar^{n-1}]$

$\Rightarrow$ $ar^2 = 4$

Product of first 5 terms $= a \cdot ar \cdot ar^2 \cdot ar^3 \cdot ar^4$

$$= a^5 r^{10} = (ar^2)^5 = (4)^5$$

Hence, the correct option is (*c*).

Q19. If 9 times the 9th term of an A.P. is equal to 13 times the 13th term, then the 22nd term of the A.P. is

(*a*) 0 (*b*) 22 (*c*) 220 (*d*) 198

Sol. $T_n = a + (n-1)d$

$\therefore$ $T_9 = a + 8d$

and $T_{13} = a + 12d$

As per the given condition

$$9[a + 8d] = 13[a + 12d]$$

$\Rightarrow$ $9a + 72d = 13a + 156d \Rightarrow -4a = 84d$

$\Rightarrow$ $a = -21d$ ...(*i*)

Now $T_{22} = a + 21d = -21d + 21d = 0$ [from eq. (*i*)]

Hence, the correct option is (*a*).

Q20. If x, $2y$, $3z$ are in A.P., where the distinct numbers x, y, z are in G.P., then the common ratio of the G.P. is

(*a*) 3 (*b*) 1/3 (*c*) 2 (*d*) 1/2

Sol. Since x, $2y$, $3z$ are in A.P.

$\therefore$ $2y - x = 3z - 2y$

$\Rightarrow$ $4y = x + 3z$ ...(*i*)

Now x, y, z are in G.P.

$\therefore$ Common ratio $r = \dfrac{y}{x} = \dfrac{z}{y}$ ...(*ii*)

$\therefore$ $y^2 = xz$

putting the value of x from eq. (*i*), we get

$$y^2 = (4y - 3z)z \Rightarrow y^2 = 4yz - 3z^2$$

$\Rightarrow$ $3z^2 - 4yz + y^2 = 0 \Rightarrow 3z^2 - 3yz - yz + y^2 = 0$

$\Rightarrow$ $3z(z - y) - y(z - y) = 0 \Rightarrow (3z - y)(z - y) = 0$

$\Rightarrow$ $3z - y = 0$ and $z - y = 0$

$\Rightarrow$ $3z = y$ and $z \neq y$

[$\because$ z and y are distinct numbers]

$\Rightarrow$ $\dfrac{z}{y} = \dfrac{1}{3} \Rightarrow r = \dfrac{1}{3}$ (from eq. (*ii*))

Hence, the correct option is (*b*).

Q21. If in an A.P., $S_n = qn^2$ and $S_m = qm^2$, where S_r denotes the sum of r terms of the A.P., then S_q equals

(*a*) $q^3/2$ (*b*) mnq (*c*) q^3 (*d*) $(m + n)q^2$

Sol. The given series is A.P. whose first term is a and common difference is d

$\therefore$ $S_n = \dfrac{n}{2}[2a + (n-1)d] = qn^2$

$\Rightarrow$ $= 2a + (n-1)d = 2qn$ ...(*i*)

$S_m = \dfrac{m}{2}[2a + (m-1)d] = qm^2$

$\Rightarrow$ $2a + (m-1)d = 2qm$ ...(*ii*)

Solving eq. (*i*) and eq. (*ii*) we get

$$\begin{array}{rcl} 2a + (m-1)d &=& 2qm \\ 2a + (n-1)d &=& 2qn \\ \hline (-) \quad (-) & & (-) \\ (m-n)d &=& 2qm - 2qn \\ (m-n)d &=& 2q(m-n) \end{array}$$

$$\therefore \quad d = 2q$$

Putting the value of d in eq. (*ii*) we get

$$2a + (m-1)\cdot 2q = 2qm \quad \Rightarrow \quad 2a = 2qm - (m-1)2q$$

$$\Rightarrow \quad 2a = 2q(m-m+1) \quad \Rightarrow \quad 2a = 2q \quad \Rightarrow \quad a = q$$

$$\therefore \quad S_q = \frac{q}{2}[2a + (q-1)d] = \frac{q}{2}[2q + (q-1)2q]$$

$$= \frac{q}{2}[2q + 2q^2 - 2q] = \frac{q}{2} \times 2q^2 = q^3$$

Hence, the correct option is (*c*).

Q22. Let S_n denote the sum of the first n terms of an A.P. if $S_{2n} = 3 \cdot S_n$ then $S_{3n} : S_n$ is equal to

(*a*) 4 (*b*) 6 (*c*) 8 (*d*) 10

Sol.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore \quad S_{2n} = \frac{2n}{2}[2a + (2n-1)d]$$

$$S_{3n} = \frac{3n}{2}[2a + (3n-1)d]$$

As per the condition of the question, we have

$$S_{2n} = 3 \cdot S_n$$

$$\Rightarrow \quad \frac{2n}{2}[2a + (2n-1)d] = 3 \cdot \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow \quad 2[2a + (2n-1)d] = 3[2a + (n-1)d]$$

$$\Rightarrow \quad 4a + (4n-2)d = 6a + (3n-3)d$$

$$\Rightarrow \quad 6a + (3n-3)d - 4a - (4n-2)d = 0$$

$$\Rightarrow \quad 2a + (3n - 3 - 4n + 2)d = 0 \quad \Rightarrow \quad 2a + (-n-1)d = 0$$

$$\Rightarrow \quad 2a - (n+1)d = 0 \quad \Rightarrow \quad 2a = (n+1)d \qquad (i)$$

$$\text{Now } S_{3n} : S_n = \frac{3n}{2}[2a + (3n-1)d] : \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{\frac{3n}{2}[2a + (3n-1)d]}{\frac{n}{2}[2a + (n-1)d]} = \frac{3[2a + (3n-1)d]}{2a + (n-1)d}$$

$$= \frac{3[(n+1)d + (3n-1)d]}{(n+1)d + (n-1)d}$$

$$= \frac{3d[n+1+3n-1]}{d(n+1+n-1)} = \frac{3[4n]}{2n} = 6$$

Hence, the correct option is (*b*).

Q23. The minimum value of $4^x + 4^{1-x}$, $x \in$ R is

(*a*) 2 (*b*) 4 (*c*) 1 (*d*) 0

Sol. We know that AM ≥ GM

$$\therefore \quad \frac{4^x + 4^{1-x}}{2} \geq \sqrt{4^x \cdot 4^{1-x}} \quad \Rightarrow \quad 4^x + 4^{1-x} \geq 2\sqrt{4^{x+1-x}}$$

$$\Rightarrow \quad 4^x + 4^{1-x} \geq 2 \cdot 2 \quad \Rightarrow \quad 4^x + 4^{1-x} \geq 4$$

Hence, the correct option is (*b*).

Q24. Let S_n denotes the sum of the cubes of the first n natural numbers and s_n denotes the sum of the first n natural numbers.

Then $\sum_{r=1}^{n} \frac{S_r}{s_n}$

(*a*) $\frac{n(n+1)(n+2)}{6}$ (*b*) $\frac{n(n+1)}{2}$

(*c*) $\frac{n^2+3n+2}{2}$ (*d*) None of these

Sol. Given that $\sum_{i=1}^{n} \frac{S_r}{s_r} = \frac{S_1}{s_1} + \frac{S_2}{s_2} + \frac{S_3}{s_3} + \cdots + \frac{S_n}{s_n}$

Let T_n be the nth term of the above series

$$\therefore \quad T_n = \frac{S_n}{s_n} = \frac{\left[\frac{n(n+1)}{2}\right]^2}{\frac{n(n+1)}{2}} = \frac{n(n+1)}{2} = \frac{n^2+n}{2}$$

Now sum of the given series

$$\sum T_n = \frac{1}{2}\sum [n^2+n] = \frac{1}{2}\left[\sum n^2 + \sum n\right]$$

$$= \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$$

$$= \frac{1}{2} \cdot \frac{n(n+1)}{2}\left[\frac{2n+1}{3} + 1\right] = \frac{n(n+1)}{4}\left[\frac{2n+1+3}{3}\right]$$

$$= \frac{n(n+1)}{4} \cdot \frac{(2n+4)}{3} = \frac{n(n+1)(n+2)}{6}$$

Hence, the correct option is (*a*).

Q25. If t_n denotes the nth term of the series 2 + 3 + 6 + 11 + 18 + …, then t_{50} is

(*a*) $49^2 - 1$ (*b*) 49^2 (*c*) $50^2 + 1$ (*d*) $49^2 + 2$

Sol. Let $S_n = 2 + 3 + 6 + 11 + 18 + \cdots + t_{50}$

Using method of difference, we get

$$S_n = 2 + 3 + 6 + 11 + 18 + \cdots + t_{50} \quad ...(i)$$

and $$S_n = 0 + 2 + 3 + 6 + 11 + \cdots + t_{49} + t_{50} \quad ...(ii)$$

Subtracting eq. (*ii*) from eq. (*i*), we get

$$0 = 2 + 1 + 3 + 5 + 7 + \cdots - t_{50} \text{ terms}$$

$$\Rightarrow \quad t_{50} = 2 + (1 + 3 + 5 + 7 + \cdots \text{ upto 49 terms})$$

$$\Rightarrow \quad t_{50} = 2 + \frac{49}{2}[2 \times 1 + (49 - 1)2] = 2 + \frac{49}{2}[2 + 96]$$

$$= 2 + \frac{49}{2} \times 98 = 2 + 49 \times 49 = 49^2 + 2$$

Hence, the correct option is (*d*).

Q26. The length of three unequal edges of a rectangular solid block are in G.P. The volume of the block is 216 cm^3 and the total surface area is 252 cm^2. The length of the largest edge is

(*a*) 12 cm (*b*) 6 cm (*c*) 18 cm (*d*) 3 cm

Sol. Let the length, breadth and height of a rectangular block be $\frac{a}{r}$, a and ar. [Since they are is G.P]

$$\therefore \quad \text{Volume} = l \times b \times h$$

$$216 = \frac{a}{r} \times a \times ar$$

$$\Rightarrow \quad a^3 = 216 \quad \Rightarrow \quad a = 6$$

Now total surface area $= 2[lb + bh + lh]$

$$252 = 2\left[\frac{a}{r} \cdot a + a \cdot ar + \frac{a}{r} \cdot ar\right] \quad \Rightarrow \quad 252 = 2\left[\frac{a^2}{r} + a^2 r + a^2\right]$$

$$\Rightarrow \quad 252 = 2a^2\left[\frac{1}{r} + r + 1\right] \quad \Rightarrow \quad 252 = 2 \times (6)^2\left[\frac{1 + r^2 + r}{r}\right]$$

$$\Rightarrow \quad 252 = 72\left[\frac{1 + r^2 + r}{r}\right] \quad \Rightarrow \quad \frac{252}{72} = \frac{1 + r + r^2}{r}$$

$$\Rightarrow \quad \frac{7}{2} = \frac{1 + r + r^2}{r} \quad \Rightarrow \quad 2 + 2r + 2r^2 = 7r$$

$$\Rightarrow \quad 2r^2 - 5r + 2 = 0 \quad \Rightarrow \quad 2r^2 - 4r - r + 2 = 0$$

$$\Rightarrow \quad 2r(r - 2) - 1(r - 2) = 0 \quad \Rightarrow \quad (r - 2)(2r - 1) = 0$$

$$\Rightarrow \quad r - 2 = 0 \quad \text{and} \quad 2r - 1 = 0$$

$$\therefore \quad r = 2, \frac{1}{2}$$

Therefore, the three edge are:

If $r = 2$ then edges are 3, 6, 12

If $r = \frac{1}{2}$ then edges are 12, 6, 3

So, the length of the longest edge = 12

Hence, the correct option is (*a*).

FILL IN THE BLANKS

Q27. If a, b and c are in G.P. then the value of $\frac{a-b}{b-c}$ is equal to ____________.

Sol. Since a, b and c are in G.P

$$\therefore \quad \frac{b}{a} = \frac{c}{b} = r \quad \text{(constant)}$$

$$\Rightarrow \quad b = ar \quad \text{and} \quad c = br \quad \Rightarrow \quad c = ar \cdot r = ar^2$$

So $$\frac{a-b}{b-c} = \frac{a-ar}{ar-ar^2} = \frac{a(1-r)}{ar(1-r)} = \frac{1}{r} = \frac{a}{b} = \frac{b}{c}$$

Hence, the correct value of the filler is $\frac{a}{b}$ or $\frac{b}{c}$

Q28. The sum of terms equidistant from the beginning and end in an A.P is equal to ____________.

Sol. Let A.P be $a, a+d, a+2d, a+3d, \ldots, a+(n-1)d$

Taking first and last term

$$a_1 + a_n = a + a + (n-1)d = 2a + (n-1)d$$

Taking second and second last term

$$a_2 + a_{n-1} = (a+d) + [a + (n-2)d] = 2a + (n-1)d = a_1 + a_n$$

Taking third from the beginning and the third from the end

$$a_3 + a_{n-2} = (a+2d) + [a + (n-3)d] = 2a + (n-1)d = a_1 + a_n$$

From the above pattern, we observe that the sum of terms equidistant from the beginning and the end in an A.P is equal to the [first term + last term]

Hence, the correct value of the filler is first term + last term.

Q29. The third term of a G.P is 4, the product of first five terms is ____________.

Sol. Given $T_3 = 4$

$$\therefore \quad ar^2 = 4 \quad \ldots(i)$$

Product of first five terms $= a \cdot ar \cdot ar^2 \cdot ar^3 \cdot ar^4$

$$= a^5 r^{10} = (ar^2)^5 = (4)^5 \quad \text{[from eq. } (i)\text{]}$$

Hence, the correct value of the fillter is $(4)^5$.

TRUE/FALSE

Q30. Two sequences can not be in both A.P and G.P together.

Sol. Let us consider a G.P, a , ar and ar^2

If it is in A.P then $ar - a \neq ar^2 - ar$

Hence, the given statement is **True**.

Q31. Every progression is a sequence but the converse i.e., every sequence is also a progression need not necessarily be true.

Sol. Let us consider a sequence of prime numbers 2, 3, 5, 7, 11, …

It is clear that this progression is a sequence but sequence is not a progression because it does not follow a specific pattern.

Here, the given statement is **True**.

Q32. Any term of an A.P (except first) is equal to half the sum of terms which are equidistant from it.

Sol. Let us consider an A.P $a, a + d, a + 2d, \ldots$

$$\therefore \quad a_2 + a_4 = a + d + a + 3d = 2a + 4d = 2a_3$$

$$\Rightarrow \quad a_3 = \frac{a_2 + a_4}{2}$$

$$\frac{a_3 + a_5}{2} = \frac{a + 2d + a + 4d}{2} = \frac{2a + 6d}{2}$$

$$\Rightarrow \quad = a + 3d = a_4$$

Hence, the given statement is **True**.

Q33. The sum or difference of two G.P's is again a G.P.

Sol. Let us consider two G.P.'s

$$a_1, a_1r_1, a_1r_1^2, a_1r_1^3 \cdots a_1r_1^{n-1}$$

and $$a_2, a_2r_2, a_2r_2^2, a_2r_2^3, \cdots a_2r_2^{n-1}$$

Now Sum of two G.Ps

$$(a_1 + a_2) + (a_1r_1 + a_2r_2) + (a_1r_1^2 + a_2r_2^2) \cdots$$

Now $$\frac{T_2}{T_1} = \frac{a_1r_1 + a_2r_2}{a_1 + a_2} \quad \text{and} \quad \frac{T_3}{T_2} = \frac{a_1r_1^2 + a_2r_2^2}{a_1r_1 + a_2r_2}$$

But $$\frac{a_1r_1 + a_2r_2}{a_1 + a_2} \neq \frac{a_1r_1^2 + a_2r_2^2}{a_1r_1 + a_2r_2}$$

Now let us consider the difference G.P's

$$(a_1 - a_2) + (a_1r_1 - a_2r_2) + (a_1r_1^2 - a_2r_2^2)$$

$$\therefore \quad \frac{T_2}{T_1} = \frac{a_1r_1 - a_2r_2}{a_1 - a_2} \quad \text{and} \quad \frac{T_3}{T_2} = \frac{a_1r_1^2 - a_2r_2^2}{a_1r_1 - a_2r_2}$$

But $$\frac{T_2}{T_1} \neq \frac{T_3}{T_2}$$

Hence, the given statement is **False**.

Q34. If the sum of n terms of a sequence is quadratic expression then it always represents an A.P

Sol. Let $\quad S_n = an^2 + bn + c \qquad$ (quadratic expression)

$$S_1 = a + b + c$$

$\therefore \quad a_1 = a + b + c$

$S_2 = 4a + 2b + c$

$a_2 = S_2 - S_1 = (4a + 2b + c) - (a + b + c) = 3a + b$

$S_3 = 9a + 3b + c$

$\Rightarrow \quad a_3 = S_3 - S_2 = (9a + 3b + c) - (4a + 2b + c) = 5a + b$

Common difference $d = a_2 - a_1 = (3a + b) - (a + b + c)$

$= 2a - c$

and $d = a_3 - a_2 = (5a + b) - (3a + b) = 2a$

Here, we observe that $a_2 - a_1 \neq a_3 - a_2$

So it does not represent an A.P

Hence, the given statement is **False**.

Q35. Match the questions given under column I with their appropriate answers given under the column II

Column I	Column II
(a) $4, 1, \frac{1}{4}, \frac{1}{16}$	(i) A.P
(b) 2, 3, 5, 7	(ii) Sequence
(c) 13, 8, 3, –2, –7	(iii) G.P

Sol. (a) $4, 1, \frac{1}{4}, \frac{1}{16}$

Here, $\frac{a_2}{a_1} = \frac{1}{4}, \frac{a_3}{a_2} = \frac{1/4}{1} = \frac{1}{4}$ and $\frac{a_4}{a_3} = \frac{1/16}{1/4} = \frac{1}{4}$

Hence it is G.P

$\therefore \quad (a) \leftrightarrow (iii)$

(b) 2, 3, 5, 7

Here $a_2 - a_1 = 3 - 2 = 1$

$a_3 - a_2 = 5 - 3 = 2$

$\therefore \quad a_2 - a_1 \neq a_3 - a_2$

Hence it is not A.P

$$\frac{a_2}{a_1} = \frac{3}{2}, \frac{a_3}{a_2} = \frac{5}{3}$$

So, $\frac{3}{2} \neq \frac{5}{3}$

So it is not G.P

Hence it is sequence

$\therefore \quad (b) \leftrightarrow (ii)$

(c) 13, 8, 3, –2, –7

Here $a_2 - a_1 = 8 - 13 = -5$

$$a_3 - a_2 = 3 - 8 = -5$$

So, $a_2 - a_1 = a_3 - a_2 = -5$

So, it is an A.P

Hence $(c) \leftrightarrow (i)$

Q36.

Column I	Column II
(a) $1^2 + 2^2 + 3^2 + \cdots + n^2$	(i) $\left[\frac{n(n+1)}{2}\right]^2$
(b) $1^3 + 2^3 + 3^3 + \cdots + n^3$	(ii) $n(n+1)$
(c) $2 + 4 + 6 + \cdots + 2n$	(iii) $\frac{n(n+1)(2n+1)}{6}$
(d) $1 + 2 + 3 + \cdots + n$	(iv) $\frac{n(n+1)}{2}$

Sol. (a) Let $S_n = 1^2 + 2^2 + 3^2 + \cdots + n^2$

$$K^3 - (K-1)^3 = 3K^2 - 3K + 1$$

For $K = 1$, $1^3 - 0^3 = 3(1)^2 - 3(1) + 1$

For $K = 2$, $2^3 - 1^3 = 3(2)^2 - 3(2) + 1$

For $K = 3$, $3^3 - 2^3 = 3(3)^2 - 3(3) + 1$

$\cdots \quad \cdots \quad \cdots$

For $K = n$, $n^3 - (n-1)^3 = 3(n)^2 - 3(n) + 1$

Adding Column wise, we get

$$n^3 - 0^3 = 3(1^2 + 2^2 + 3^2 + \cdots + n^2) - 3(1 + 2 + 3 + \cdots + n) + n$$

$$\Rightarrow \quad n^3 = 3 \cdot S_n - \frac{3n(n+1)}{2} + n$$

$$\Rightarrow \quad n^3 + \frac{3n(n+1)}{2} - n = 3 \cdot S_n$$

$$\Rightarrow \quad \frac{2n^3 + 3n^2 + 3n - 2n}{2} = 3 \cdot S_n$$

$$\Rightarrow \quad 6 \cdot S_n = 2n^3 + 3n^2 + n$$

$$\Rightarrow \quad 6 \cdot S_n = n(2n^2 + 3n + 1)$$

$$\Rightarrow \quad 6 \cdot S_n = n[2n^2 + 2n + n + 1]$$

$$\Rightarrow \quad 6 \cdot S_n = n(n+1)(2n+1)$$

$$\Rightarrow \quad S_n = \frac{n(n+1)(2n+1)}{6}$$

Here $(a) \leftrightarrow (iii)$.

(b) Let $S_n = 1^3 + 2^3 + 3^3 + \cdots + n^3$

$$K^4 - (K-1)^4 = 4K^3 - 6K^2 + 4K - 1$$

For K = 1 $\quad 1^4 - 0^4 = 4(1)^3 - 6(1)^2 + 4(1) - 1$

For K = 2 $\quad 2^4 - 1^4 = 4(2)^3 - 6(2)^2 + 4(2) - 1$

For K = 3 $\quad 3^4 - 2^4 = 4(3)^3 - 6(3)^2 + 4(3) - 1$

$\ldots \quad \ldots \quad \ldots$

For K = n $\quad n^4 - (n-1)^4 = 4(n)^3 - 6(n^2) + 4(n) - 1$

Adding column wise, we get

$$\Rightarrow \quad n^4 - 0^4 = 4(1^3 + 2^3 + 3^3 + \cdots n^3) - 6(1^2 + 2^2 + 3^2 + \cdots + n^2) + 4(1 + 2 + 3 + \ldots n) - n$$

$$\Rightarrow \quad n^4 = 4 \cdot S_n - \frac{6n(n+1)(2n+1)}{6} + \frac{4n(n+1)}{2} - n$$

$$\Rightarrow \quad n^4 = 4 \cdot S_n - n(n+1)(2n+1) + 2n(n+1) - n$$

$$\Rightarrow \quad n^4 + n(n+1)(2n+1) - 2n(n+1) + n = 4 \cdot S_n$$

$$\Rightarrow \quad n[n^3 + (n+1)(2n+1) - 2n - 2 + 1] = 4 \cdot S_n$$

$$\Rightarrow \quad n[n^3 + 2n^2 + 3n + 1 - 2n - 1] = 4 \cdot S_n$$

$$\Rightarrow \quad n[n^3 + 2n^2 + n] = 4 \cdot S_n$$

$$\Rightarrow \quad \frac{n^2(n^2 + 2n + 1)}{4} = S_n$$

$$\Rightarrow \quad \frac{n^2(n+1)^2}{4} = S_n$$

$$\therefore \quad S_n = \left[\frac{n(n+1)}{2}\right]^2$$

Hence $(b) \leftrightarrow (i)$

(c) Let $S_n = 2 + 4 + 6 + \ldots + 2n$

$= 2(1 + 2 + 3 + \ldots + n)$

$= 2\dfrac{n(n+1)}{2}$

$= n(n+1)$

Hence $(c) \leftrightarrow (ii)$

(d) Let $S_n = 1 + 2 + 3 + \cdots + n = \dfrac{n(n+1)}{2}$

Hence $(d) \leftrightarrow (iv)$