EXERCISE

SHORT ANSWER TYPE QUESTIONS

Q1. Locate the following points:

(*i*) (1, – 1, 3) (*ii*) (– 1, 2, 4) (*iii*) (– 2, – 4, – 7)
(*iv*) (– 4, 2, – 5)

Sol. (*i*) Location of P(1, – 1, 3) = $(x, -y, z)$ = IV octant
(*ii*) Location of Q(– 1, 2, 4) = $(-x, y, z)$ = II octant
(*iii*) Location of R(– 2, – 4, – 7) = $(-x, -y, -z)$ = VII octant
(*iv*) Location of S(– 4, 2, – 5) = $(-x, y, -z)$ = VI octant

Q2. Name the octant in which each of the following points lie.

(*i*) (1, 2, 3) (*ii*) (4, – 2, 3) (*iii*) (4, – 2, – 5)
(*iv*) (4, 2, – 5) (*v*) (– 4, 2, 5) (*vi*) (– 3, – 1, 6)
(*vii*) (2, – 4, – 7) (*viii*) (– 4, 2, – 5)

Sol. (*i*) Point (1, 2, 3) lies in I octant
(*ii*) Point (4, – 2, 3) lies in IV octant
(*iii*) Point (4, – 2, – 5) lies in VIII octant
(*iv*) Point (4, 2, – 5) lies in V octant
(*v*) Point (– 4, 2, 5) lies in II octant
(*vi*) Point (– 3, – 1, 6) lies in III octant
(*vii*) Point (2, – 4, – 7) lies in VIII octant
(*viii*) Point (– 4, 2, – 5) lies in VI octant

Q3. Let A, B, C be the feet of perpendiculars from a point P on x, y, z-axis respectively. Find the coordinates of A, B and C in each of the following where the P is

(*i*) (3, 4, 2) (*ii*) (– 5, 3, 7) (*iii*) (4, – 3, – 5)

Sol. The coordinates of A, B and C are
(*i*) A(3, 0, 0), B(0, 4, 0) and C(0, 0, 2)
(*ii*) A(– 5, 0, 0), B(0, 3, 0) and C(0, 0, 7)
(*iii*) A(4, 0, 0), B(0, – 3, 0) and C(0, 0, – 5)

Q4. Let A, B, C be the feet of perpendicular from a point P on the xy, yz and zx planes respectively. Find the coordinates of A, B, C in each of the following where point P is

(*i*) (3, 4, 5) (*ii*) (– 5, 3, 7) (*iii*) (4, – 3, – 5)

Sol. The coordinates of A, B and C are
(*i*) A(3, 4, 0), B(0, 4, 5) and C(3, 0, 5)

(*ii*) A(– 5, 3, 0), B(0, 3, 7) and C(– 5, 0, 7)

(*iii*) A(4, – 3, 0), B(0, – 3, – 5) and C(4, 0, – 5)

Q5. How far apart are the points (2, 0, 0) and (– 3, 0, 0)?

Sol. Given points are (2, 0, 0) and (– 3, 0, 0)

$\therefore$ Distance between the given points

$$= \sqrt{(2+3)^2 + (0-0)^2 + (0-0)^2} = \sqrt{25} = 5$$

Hence, the required distance = 5.

Q6. Find the distance from the origin to (6, 6, 7).

Sol. Coordinates of the origin are (0, 0, 0)

$\therefore$ Distance from (0, 0, 0) to (6, 6, 7)

$$= \sqrt{(6-0)^2 + (6-0)^2 + (7-0)^2}$$

$$= \sqrt{36+36+49} = \sqrt{121}$$

= 11 units.

Hence, the required distance = 11 units.

Q7. Show that, if $x^2 + y^2 = 1$, then the point $\left(x, y, \sqrt{1-x^2-y^2}\right)$ is at a distance unit from the origin.

Sol. Given point is $\left(x, y, \sqrt{1-x^2-y^2}\right)$

$\therefore$ Distance between the origin and the point is

$$= \sqrt{(x-0)^2 + (y-0)^2 + \left(\sqrt{1-x^2-y^2} - 0\right)^2}$$

$= \sqrt{1} = 1$. Hence, proved.

Q8. Show that the points A(1, – 1, 3), B(2, – 4, 5) and C(5, – 13, 11) are collinear.

Sol. Given points are A(1, – 1, 3), B(2, – 4, 5) and C(5, – 13, 11)

$$AB = \sqrt{(2-1)^2 + (-4+1)^2 + (5-3)^2}$$

$$= \sqrt{1+9+4} = \sqrt{14}$$

$$BC = \sqrt{(5-2)^2 + (-13+4)^2 + (11-5)^2}$$

$$= \sqrt{9+81+36} = \sqrt{126} = 3\sqrt{14}$$

$$AC = \sqrt{(5-1)^2 + (-13+1)^2 + (11-3)^2}$$

$$= \sqrt{16+144+64} = \sqrt{224} = 4\sqrt{14}$$

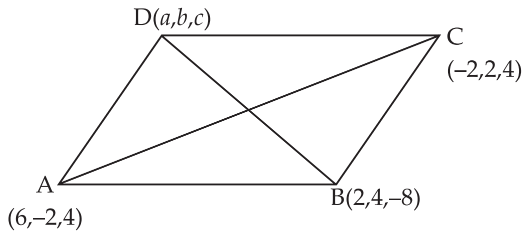
Here we observe that $\sqrt{14} + 3\sqrt{14} = 4\sqrt{14}$

So AB + BC = AC.

Hence, the given points are collinear.

Q9. Three conjugative vertices of a parallelogram ABCD are A(6, – 2, 4), B(2, 4, – 8) and C(– 2, 2, 4). Find the coordinates of the fourth vertex.

Sol. Let the coordinates of the fourth vertex be (a, b, c)



We know that the diagonals of a parallelogram bisect each other.

$\therefore$ Mid-point of diagonal AC $= \left(\frac{6-2}{2}, \frac{-2+2}{2}, \frac{4+4}{2}\right)$

$= (2, 0, 4)$

and the mid-point of diagonal BD

$$= \left(\frac{a+2}{2}, \frac{b+4}{2}, \frac{c-8}{2}\right)$$

$$\therefore \qquad \frac{a+2}{2} = 2 \Rightarrow a = 2$$

$$\frac{b+4}{2} = 0 \Rightarrow b = -4$$

$$\text{and} \qquad \frac{c-8}{2} = 4 \Rightarrow c = 16$$

Hence, the required coordinates are (2, – 4, 16).

Q10. Show that the ΔABC with vertices A(0, 4, 1), B(2, 3, – 1) and C(4, 5, 0)is right angled.

Sol. Given vertices are A(0, 4, 1), B(2, 3, – 1) and C(4, 5, 0)

$$AB = \sqrt{(2-0)^2 + (3-4)^2 + (-1-1)^2}$$

$$= \sqrt{4+1+4} = \sqrt{9} = 3$$

$$BC = \sqrt{(4-2)^2 + (5-3)^2 + (0+1)^2}$$

$$= \sqrt{4+4+1} = \sqrt{9} = 3$$

$$AC = \sqrt{(4-0)^2 + (5-4)^2 + (0-1)^2}$$

$$= \sqrt{16+1+1} = \sqrt{18}$$

$\because \quad (3)^2 + (3)^2 = \left(\sqrt{18}\right)^2$.

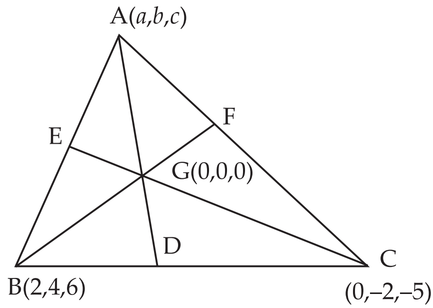
So $AB^2 + BC^2 = AC^2$

Hence, ΔABC is a right angled triangle.

Q11. Find the third vertex of triangle whose centroid is origin and two vertices are (2, 4, 6) and (0, – 2, – 5).

Sol. Let the coordinates of the third vertex i.e. A be (a, b, c).

Since the centroid is at origin *i.e.* (0, 0, 0)



$$\therefore \quad 0 = \frac{a+2+0}{3} \Rightarrow a = -2$$

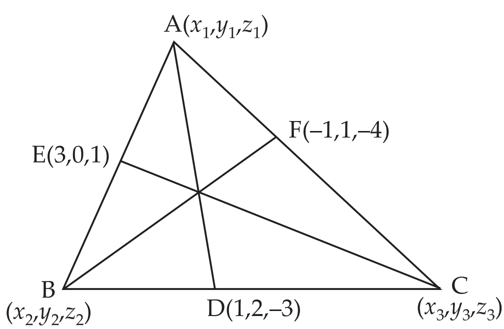
$$0 = \frac{b+4-2}{3} \Rightarrow b = -2$$

$$\text{and} \quad 0 = \frac{c+6-5}{3} \Rightarrow c = -1$$

Hence, the required coordinates are (– 2, – 2, – 1).

Q12. Find the centroid of a triangle, the mid-point of whose sides are D(1, 2, – 3), E(3, 0, 1) and F(– 1, 1, – 4).

Sol. Let the coordinates of the vertices of ΔABC be $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$.



Mid-point of BC = (1, 2, – 3)

$\therefore \quad 1 = \dfrac{x_2 + x_3}{2} \Rightarrow x_2 + x_3 = 2$ ...(*i*)

$2 = \dfrac{y_2 + y_3}{2} \Rightarrow y_2 + y_3 = 4$ ...(*ii*)

$-3 = \dfrac{z_2 + z_3}{2} \Rightarrow z_2 + z_3 = -6$ ...(*iii*)

Mid-point of AB = (3, 0, 1)

$\therefore \quad 3 = \dfrac{x_1 + x_2}{2} \Rightarrow x_1 + x_2 = 6$ ...(*iv*)

$0 = \dfrac{y_1 + y_2}{2} \Rightarrow y_1 + y_2 = 0$ ...(*v*)

$1 = \dfrac{z_1 + z_2}{2} \Rightarrow z_1 + z_2 = 2$ ...(*vi*)

Similarly, mid-point of AC = (– 1, 1, – 4)

$\therefore \quad -1 = \dfrac{x_1 + x_3}{2} \Rightarrow x_1 + x_3 = -2$ ...(*vii*)

$1 = \dfrac{y_1 + y_3}{2} \Rightarrow y_1 + y_3 = 2$ ...(*viii*)

$-4 = \dfrac{z_1 + z_3}{2} \Rightarrow z_1 + z_3 = -8$ ...(*ix*)

Adding eq. (*i*), (*iv*) and (*vii*) we get,

$$2x_1 + 2x_2 + 2x_3 = 2 + 6 - 2 = 6$$

$\Rightarrow \quad x_1 + x_2 + x_3 = 3$

$\Rightarrow \quad 6 + x_3 = 3 \Rightarrow x_3 = -3$ [from eq. (*iv*)]

$\Rightarrow \quad x_1 + 2 = 3 \Rightarrow x_1 = 1$ [from eq. (*i*)]

$\Rightarrow \quad x_2 - 2 = 3 \Rightarrow x_2 = 5$ [from eq. (*vii*)]

So, $x_1 = 1$, $x_2 = 5$ and $x_3 = -3$.

Similarly, Adding (*ii*), (*v*) and (*viii*) we get

$$2(y_1 + y_2 + y_3) = 4 + 0 + 2 = 6$$

$\therefore \quad y_1 + y_2 + y_3 = 3$

$y_1 + 4 = 3 \Rightarrow y_1 = -1$

$0 + y_3 = 3 \Rightarrow y_3 = 3$

$y_2 + 2 = 3 \Rightarrow y_2 = 1$

So, $y_1 = -1$, $y_2 = 1$, $y_3 = 3$

Adding (*iii*), (*vi*) and (*ix*) we have

$$2(z_1 + z_2 + z_3) = -6 + 2 - 8 = -12$$

$\therefore \quad z_1 + z_2 + z_3 = -6$

$z_1 - 6 = -6 \Rightarrow z_1 = 0$ [from eq. (*iii*)]

$2 + z_3 = -6 \Rightarrow z_3 = -8$ [from eq. (*vi*)]

and $z_2 - 8 = -6 \Rightarrow z_2 = 2$

$\therefore z_1 = 1, z_2 = 1, z_3 = -8.$

So, the points are A(1, – 1, 0), B(5, 1, 2) and C(– 3, 3, – 8).

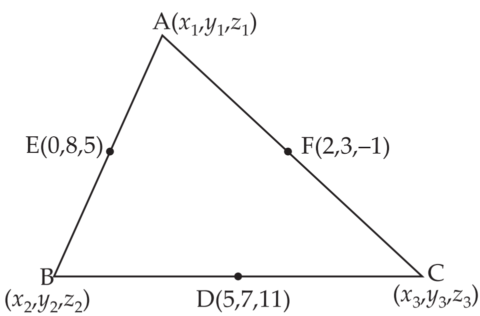
$\therefore$Centroid of the triangle

$$G = \left(\frac{1+5-3}{3}, \frac{-1+1+3}{3}, \frac{0+2-8}{3}\right) = (1, 1, -2)$$

Hence, the required coordinates = (1, 1, – 2).

Q13. The mid-points of the sides of a triangle are (5, 7, 11), (0, 8, 5) and (2, 3, – 1). Find the vertices.

Sol. Let the coordinates of the vertices be $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ respectively.



Since D(5, 7, 11) is the mid-point of BC

$$\therefore \quad 5 = \frac{x_2 + x_3}{2} \Rightarrow x_2 + x_3 = 10 \quad ...(i)$$

$$7 = \frac{y_2 + y_3}{2} \Rightarrow y_2 + y_3 = 14 \quad ...(ii)$$

$$11 = \frac{z_2 + z_3}{2} \Rightarrow z_2 + z_3 = 22 \quad ...(iii)$$

E(0, 8, 5) is the mid-point of AB

$$\therefore \quad 0 = \frac{x_1 + x_2}{2} \Rightarrow x_1 + x_2 = 0 \quad ...(iv)$$

$$8 = \frac{y_1 + y_2}{2} \Rightarrow y_1 + y_2 = 16 \quad ...(v)$$

$$5 = \frac{z_1 + z_2}{2} \Rightarrow z_1 + z_2 = 10 \quad ...(vi)$$

Similarly, F(2, 3, – 1) is the mid-point of AC

$$\therefore \quad 2 = \frac{x_1 + x_3}{2} \Rightarrow x_1 + x_3 = 4 \quad \text{...}(vii)$$

$$3 = \frac{y_1 + y_3}{2} \Rightarrow y_1 + y_3 = 6 \quad \text{...}(viii)$$

$$\text{and} \quad -1 = \frac{z_1 + z_3}{2} \Rightarrow z_1 + z_3 = -2 \quad \text{...}(ix)$$

Adding eq. (*i*), (*iv*) and (*vii*) we get

$$2(x_1 + x_2 + x_3) = 10 + 0 + 4$$

$$\Rightarrow \quad x_1 + x_2 + x_3 = 7 \quad \text{...}(x)$$

Subtracting (*i*) from (*x*) we get,

$$x_1 = 7 - 10 = -3$$

Subtracting eq. (*iv*) from (*x*) we get

$$x_3 = 7 - 0 = 7$$

Subtracting eq. (*vii*) from (*x*) we get

$$x_2 = 7 - 4 = 3$$

Adding eq. (*ii*), (*v*) and (*viii*) we get

$$2(y_1 + y_2 + y_3) = (14 + 16 + 6)$$

$$\Rightarrow \quad y_1 + y_2 + y_3 = 18 \quad \text{...}(xi)$$

Subtracting eq. (*ii*) from (*xi*) we get

$$y_1 = 18 - 14 = 4$$

Subtracting eq. (*v*) from (*xi*) we get

$$y_3 = 18 - 16 = 2$$

Subtracting eq. (*viii*) from (*xi*) we get

$$y_2 = 18 - 6 = 12$$

Now adding eq. (*iii*), (*vi*) and (*ix*) we get

$$2(z_1 + z_2 + z_3) = 22 + 10 - 2$$

$$\Rightarrow \quad z_1 + z_2 + z_3 = 15 \quad \text{...}(xii)$$

Subtracting eq. (*iii*) from (*xii*) we get

$$z_1 = 15 - 22 = -7$$

Subtracting eq. (*vi*) from (*xii*), we get

$$z_3 = 15 - 10 = 5$$

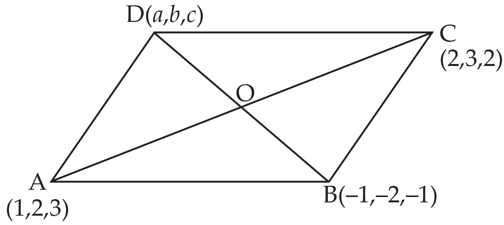
Subtracting eq. (*ix*) from (*xii*) we get

$$z_2 = 15 + 2 = 17$$

So, the required coordinates are
$A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$.
i.e. A(– 3, 4, – 7), B(3, 12, 17) and C(7, 2, 5).

Q14. Three vertices of a parallelogram ABCD are A(1, 2, 3), B(– 1, – 2, – 1) and C(2, 3, 2) find the fourth vertex D.

Sol. Let the coordinates of D be (a, b, c).



We know that the diagonals of a parallelogram bisect each other.

$\therefore$ Mid-point of AC i.e. $O = \left(\frac{1+2}{2}, \frac{2+3}{2}, \frac{3+2}{2}\right)$

$$= \left(\frac{3}{2}, \frac{5}{2}, \frac{5}{2}\right)$$

Mid-point of BD i.e. $O = \left(\frac{a-1}{2}, \frac{b-2}{2}, \frac{c-1}{2}\right)$

Equating the corresponding coordinate, we have

$$\frac{a-1}{2} = \frac{3}{2} \Rightarrow a = 4$$

$$\frac{b-2}{2} = \frac{5}{2} \Rightarrow b = 7$$

and $$\frac{c-1}{2} = \frac{5}{2} \Rightarrow c = 6$$

Hence, the coordinates of D = (4, 7, 6).

Q15. Find the coordinate of the points which trisect the line segment joining the points A(2, 1, – 3) and B(5, – 8, 3).

Sol. Let C and D be the points which divide the given line AB into three equal parts.

Here AC : CB = 1 : 2

Let (x_1, y_1, z_1) be the coordinates of C

$\therefore$ $$x_1 = \frac{1 \times 5 + 2 \times 2}{1+2} = 3$$

$$y_1 = \frac{1 \times -8 + 2 \times 1}{1+2} = -2$$

$$z_1 = \frac{1 \times 3 + 2 \times -3}{1+2} = -1$$

So C = (3, – 2, – 1).

Now D is the mid-point = CB

Let the coordinates of D be (x_2, y_2, z_2)

$$\therefore \quad x_2 = \frac{3+5}{2} = 4$$

$$y_2 = \frac{-8-2}{2} = -5$$

and $$z_2 = \frac{3-1}{2} = 1$$

So, D = (4, – 5, 1).

Hence, the required coordinates are C(3, – 2, – 1) and D(4, – 5, 1).

Q16. If the origin is the centroid of a triangle ABC having vertices A(a, 1, 3), B(– 2, b, – 5) and C(4, 7, c) find the values of a, b, c.

Sol. Coordinates of the centroid G = (0, 0, 0)

$$\therefore \quad 0 = \frac{x_1 + x_2 + x_3}{3} \Rightarrow 0 = \frac{a-2+4}{3} \Rightarrow a = -2$$

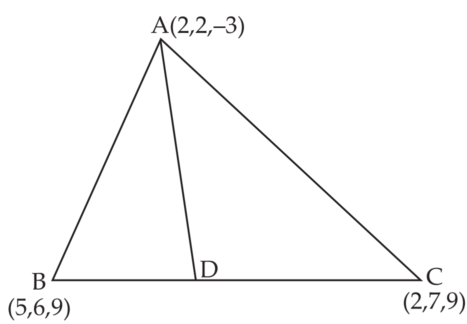
$$0 = \frac{y_1 + y_2 + y_3}{3} \Rightarrow 0 = \frac{1+b+7}{3} \Rightarrow b = -8$$

and $$0 = \frac{z_1 + z_2 + z_3}{3} \Rightarrow 0 = \frac{3-5+c}{3} \Rightarrow c = 2$$

Hence, the required values are $a = -2$, $b = -8$ and $c = 2$.

Q17. Let A(2, 2, – 3), B(5, 6, 9) and C(2, 7, 9) be the vertices of a triangle. The internal bisector of the angle A meets BC at the point D. Find the coordinates of D.

Sol. Given that AD is the internal bisector of ∠A



$$\therefore \qquad \frac{AB}{AC} = \frac{BD}{DC}$$

$$AB = \sqrt{(5-2)^2 + (6-2)^2 + (9+3)^2}$$
$$= \sqrt{9+16+144} = \sqrt{169} = 13$$
$$AC = \sqrt{(2-2)^2 + (7-2)^2 + (9+3)^2}$$
$$= \sqrt{0+25+144} = 13$$

$$\therefore \qquad \frac{AB}{AC} = \frac{BD}{DC} = \frac{13}{13} \quad \Rightarrow \quad BD = DC$$

$\Rightarrow$ D is the mid-point of BC

$$\therefore \qquad \text{Coordinates of D} = \left(\frac{5+2}{2}, \frac{6+7}{2}, \frac{9+9}{2}\right)$$
$$= \left(\frac{7}{2}, \frac{13}{2}, 9\right)$$

Hence, the required coordinates are $\left(\frac{7}{2}, \frac{13}{2}, 9\right)$.

LONG ANSWER TYPE QUESTIONS

Q18. Show that the three points A(2, 3, 4), B(– 1, 2, – 3) and C(– 4, 1, – 10) are collinear and find the ratio in which C divides AB.

Sol. Given points are A(2, 3, 4), B(– 1, 2, – 3) and C(– 4, 1, – 10)

$$AB = \sqrt{(2+1)^2 + (3-2)^2 + (4+3)^2}$$
$$= \sqrt{9+1+49} = \sqrt{59}$$
$$BC = \sqrt{(-1+4)^2 + (2-1)^2 + (-3+10)^2}$$
$$= \sqrt{9+1+49} = \sqrt{59}$$
$$AC = \sqrt{(2+4)^2 + (3-1)^2 + (4+10)^2}$$
$$= \sqrt{36+4+196} = \sqrt{236} = 2\sqrt{59}$$

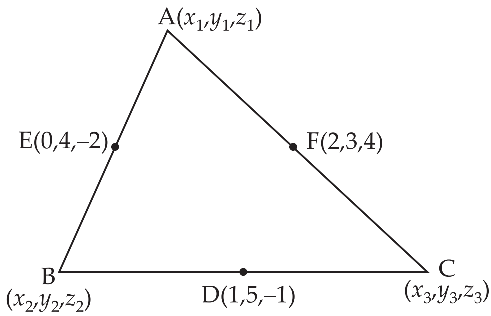
$$\therefore \qquad AB + BC = AC$$
$$\sqrt{59} + \sqrt{59} = 2\sqrt{59}$$

Hence, A, B and C are collinear and AC : BC = $2\sqrt{59} : \sqrt{59} = 2:1$

Hence, C divides AB is 2 : 1 externally.

Q19. The mid-point of the sides of a triangle are (1, 5, – 1), (0, 4, – 2) and (2, 3, 4). Find its vertices. Also find the centroid of the triangle.

Sol. Let the vertices of the given triangle be
$A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$.



D is the mid-point of BC

$$\therefore \quad 1 = \frac{x_2 + x_3}{2} \Rightarrow x_2 + x_3 = 2 \quad ...(i)$$

$$5 = \frac{y_2 + y_3}{2} \Rightarrow y_2 + y_3 = 10 \quad ...(ii)$$

$$-1 = \frac{z_2 + z_3}{2} \Rightarrow z_2 + z_3 = -2 \quad ...(iii)$$

E is the mid-point of AB

$$\therefore \quad 0 = \frac{x_1 + x_2}{2} \Rightarrow x_1 + x_2 = 0 \quad ...(iv)$$

$$4 = \frac{y_1 + y_2}{2} \Rightarrow y_1 + y_2 = 8 \quad ...(v)$$

$$-2 = \frac{z_1 + z_2}{2} \Rightarrow z_1 + z_2 = -4 \quad ...(vi)$$

F is the mid-point of AC

$$\therefore \quad 2 = \frac{x_1 + x_3}{2} \Rightarrow x_1 + x_3 = 4 \quad ...(vii)$$

$$3 = \frac{y_1 + y_3}{2} \Rightarrow y_1 + y_3 = 6 \quad ...(viii)$$

$$4 = \frac{z_1 + z_3}{2} \Rightarrow z_1 + z_3 = 8 \quad ...(ix)$$

Adding eq. (*i*), (*iv*) and (*vii*) we get

$$2(x_1 + x_2 + x_3) = 2 + 0 + 4$$

$$\therefore \quad x_1 + x_2 + x_3 = 3 \quad ...(x)$$

Adding (*ii*), (*v*) and (*viii*) we get

$$2(y_1 + y_2 + y_3) = 10 + 8 + 6$$

$$\Rightarrow \quad y_1 + y_2 + y_3 = 12 \quad ...(xi)$$

Adding (*iii*), (*vi*) and (*ix*) we get

$$2(z_1 + z_2 + z_3) = -2 - 4 + 8$$

$$\therefore \quad z_1 + z_2 + z_3 = 1 \quad ...(xii)$$

Subtracting eq. (*i*) from (*x*) we get

$$x_1 = 3 - 2 = 1 \Rightarrow x_1 = 1$$

Subtracting eq. (*ii*) from (*xi*) we get

$$y_1 = 12 - 10 = 2 \Rightarrow y_1 = 2$$

Subtracting eq. (*iii*) from (*xii*) we get

$$z_1 = 1 - (-2) = 3 \Rightarrow z_1 = 3$$

Hence, the coordinates of A = (1, 2, 3)

Subtracting eq. (*iv*) from (*x*) we get

$$x_3 = 3 - 0 = 3 \Rightarrow x_3 = 3$$

Subtracting eq. (*v*) from (*xi*) we get

$$y_3 = 12 - 8 = 4 \Rightarrow y_3 = 4$$

Subtracting eq. (*vi*) from (*xii*) we get

$$z_3 = 1 - (-4) = 5 \Rightarrow z_3 = 5$$

Here the coordinates of C = (3, 4, 5)

Subtracting eq. (*vii*) from (*x*) we get

$$x_2 = 3 - 4 = -1 \Rightarrow x_2 = -1$$

Subtracting eq. (*viii*) from (*xi*) we get

$$y_2 = 12 - 6 = 6 \Rightarrow y_2 = 6$$

Subtracting eq. (*ix*) from (*xii*) we get

$$z_2 = 1 - 8 = -7 \Rightarrow z_2 = -7$$

Here, the coordinates of B(– 1, 6, – 7)

Hence, the required coordinates are

A(1, 2, 3), B(– 1, 6, – 7) and C(3, 4, 5)

and Centroid $G = \left(\frac{1-1+3}{3}, \frac{2+6+4}{3}, \frac{3-7+5}{3}\right)$

$$= \left(1, 4, \frac{1}{3}\right).$$

Q20. Prove that the points (0, – 1, – 7), (2, 1, – 9) and (6, 5, – 13) are collinear. Find the ratio in which the first point divides the join of the other two.

Sol. Let the given points are A(0, – 1, – 7), B(2, 1, – 9) and C(6, 5, – 13)

$$AB = \sqrt{(2-0)^2 + (1+1)^2 + (-9+7)^2}$$

$$= \sqrt{4+4+4} = \sqrt{12} = 2\sqrt{3}$$

$$BC = \sqrt{(6-2)^2 + (5-1)^2 + (-13+9)^2}$$

$$= \sqrt{16+16+16} = \sqrt{48} = 4\sqrt{3}$$

$$AC = \sqrt{(6-0)^2 + (5+1)^2 + (-13+7)^2}$$
$$= \sqrt{36+36+36} = \sqrt{108} = 6\sqrt{3}$$

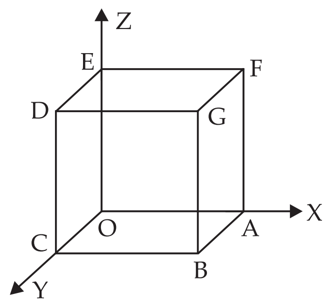
$$2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$$

i.e. $AB + BC = AC$

$\therefore$ $AB : AC = 2\sqrt{3} : 6\sqrt{3} = 1 : 3$

Hence, point A divides B and C in 1 : 3 externally.

Q21. What are the coordinates of the vertices of a cube whose edge is 2 units, one of whose vertices coincides with the origin and the three edges passing through the origin, coincides with the positive direction of the axes through the origin?



Sol. Given that each edge of the cuboid is 2 units.

$\therefore$ Coordinates of the vertices are

A(2, 0, 0), B(2, 2, 0), C(0, 2, 0), D(0, 2, 2), E(0, 0, 2), F(2, 0, 2), G(2, 2, 2) and O(0, 0, 0).

OBJECTIVE TYPE QUESTIONS

Q22. The distance of point P(3, 4, 5) from the *yz*-plane is

(*a*) 3 units (*b*) 4 units

(*c*) 5 units (*d*) 550

Sol. Given point is P(3, 4, 5)

$\therefore$ Distance of P from *yz*-plane

$$= \sqrt{(0-3)^2 + (4-4)^2 + (5-5)^2}$$
$$= \sqrt{9} = 3 \text{ units}$$

Hence, the correct option is (*a*).

Q23. What is the length of foot of perpendicular drawn from the point P(3, 4, 5) on *y*-axis?

(*a*) $\sqrt{41}$ (*b*) $\sqrt{34}$

(*c*) 5 (*d*) None of these

Sol. On *y*-axis, $x = 0$ and $z = 0$ and given point P(3, 4, 5)

$\therefore$ The point A is (0, 4, 0)

$\therefore$ $$PA = \sqrt{(0-3)^2 + (4-4)^2 + (0-5)^2}$$
$$= \sqrt{9+0+25} = \sqrt{34}$$

Hence, the correct option is (*b*).

Q24. Distance of the point (3, 4, 5) from the origin (0, 0, 0) is

(*a*) $\sqrt{50}$ (*b*) 3

(*c*) 4 (*d*) 5

Sol. Given points A(3, 4, 5) and the given O(0, 0, 0)

$$\therefore \quad OA = \sqrt{(3-0)^2 + (4-0)^2 + (5-0)^2}$$

$$= \sqrt{9+16+25} = \sqrt{50}$$

Hence, the correct option is (*a*).

Q25. If the distance between the points (a, 0, 1) and (0, 1, 2) is $\sqrt{27}$, then the value of 'a' is

(*a*) 5 (*b*) ± 5

(*c*) – 5 (*d*) None of these

Sol. Let the given points be A(a, 0, 1) and B(0, 1, 2)

$$\therefore \quad AB = \sqrt{(a-0)^2 + (0-1)^2 + (1-2)^2}$$

$$\sqrt{27} = \sqrt{a^2+1+1}$$

Squaring both sides, we get

$$27 = a^2 + 2 \quad \Rightarrow \quad a^2 = 25 \quad \therefore a = \pm 5$$

Hence, the correct option is (*b*).

Q26. x-axis is the intersection of two planes.

(*a*) xy and xz (*b*) yz and zx

(*c*) xy and yz (*d*) None of these

Sol. We know that on the xy and xz-planes, the line of intersection is x-axis.

Hence, the correct option is (*a*).

Q27. Equation of y-axis is considered as

(*a*) $x = 0, y = 0$ (*b*) $y = 0, z = 0$

(*c*) $z = 0, x = 0$ (*d*) None of these

Sol. On y-axis, $x = 0$ and $z = 0$

Hence, the correct option is (*c*).

Q28. The point (– 2, – 3, – 4) lies in the

(*a*) first octant (*b*) seventh octant

(*c*) second octant (*d*) eighth octant

Sol. The point (– 2, – 3, – 4) lies in seventh octant.

Hence, the correct option is (*b*).

Q29. A plane is parallel to yz-plane, so it is perpendicular to

(*a*) x-axis (*b*) y-axis

(*c*) z-axis (*d*) None of these

Sol. Any plane parallel to yz-plane is perpendicular to x-axis.

Hence, the correct option is (*a*).

Q30. The locus of a point for which $y = 0, z = 0$ is

(*a*) equation of x-axis (*b*) equation of y-axis

(*c*) equation of z-axis (*d*) None of these

Sol. We know that one equation of x-axis, $y = 0, z = 0$

Hence, the locus of the point is equation of x-axis.

So, the correct option is (*a*).

Q31. The locus of a point for which $x = 0$ is

(*a*) xy-plane (*b*) yz-plane

(*c*) zx-plane (*d*) None of these

Sol. On the yz-plane, $x = 0$

Hence, the locus of the point is yz-plane.

So, the correct option is (*b*).

Q32. If a parallelopiped is formed by planes drawn through the points (5, 8, 10) and (3, 6, 8) parallel to the coordinate planes, then the length of diagonal of the parallelopiped is

(*a*) $2\sqrt{3}$ (*b*) $3\sqrt{2}$

(*c*) $\sqrt{2}$ (*d*) $\sqrt{3}$

Sol. Given points are A(5, 8, 10) and B(3, 6, 8)

$$\therefore \quad AB = \sqrt{(5-3)^2 + (8-6)^2 + (10-8)^2}$$

$$= \sqrt{4+4+4} = \sqrt{12} = 2\sqrt{3}$$

Hence, the correct option is (*a*).

Q33. L is the foot of perpendicular drawn from a point P(3, 4, 5) on xy-plane. The coordinate of point L are

(*a*) (3, 0, 0) (*b*) (0, 4, 5)

(*c*) (3, 0, 5) (*d*) None of these

Sol. We know that on xy-plane, $z = 0$.

So, the coordinate of the point L are (3, 4, 0).

Hence, the correct option is (*d*).

Q34. L is the foot of the perpendicular drawn from point (3, 4, 5) on X-axis. The coordinates of L are

(*a*) (3, 0, 0) (*b*) (0, 4, 0)

(*c*) (0, 0, 5) (*d*) None of these

Sol. We know that x-axis, $y = 0$ and $z = 0$.

So, the required coordinates are (3, 0, 0).

Hence, the correct option is (*a*).

Fill in the Blanks in Each of the Exercises from 35 to 49.

Q35. The three axes OX, OY and OZ determine ....

Sol. The three axes OX, OY and OZ determine three coordinate planes.

Hence, the filler value is **three coordinate planes**.

Q36. The three planes determine a rectangular parallelopiped which has .... of rectangular faces.

Sol. Three pairs.
Hence, the value of the filler is **three pairs**.

Q37. The coordinates of a point are the perpendicular distance from the .... on respective axes.

Sol. Given points.
Hence, the value of the filler is **given points**.

Q38. The three coordinates planes divide the space into .... parts.

Sol. Eight.
Hence, the values of the filler is **eight**.

Q39. If a point P lies in yz-plane, then the coordinates of a point on yz-plane is of the form ....

Sol. We know that on yz-plane, $x = 0$
So, the coordinates of the required point is $(0, y, z)$.
Hence, the value of the filler is $\mathbf{(0, y, z)}$.

Q40. The equation of yz-plane is ....

Sol. The equation of yz-plane is $x = 0$.
Hence, the value of the filler is $\mathbf{x = 0}$.

Q41. If the point P lies on z-axis, then coordinates of P are of the form ....

Sol. On the z-axis, $x = 0$ and $y = 0$.
$\therefore$ The required coordinate is in the form of $(0, 0, z)$.
Hence, the value of the filler is $\mathbf{(0, 0, z)}$.

Q42. The equation of z-axis are ....

Sol. The equation of z-axis are, $x = 0$ and $y = 0$.
Hence, the value of the filler is $\mathbf{x = 0}$ **and** $\mathbf{y = 0}$.

Q43. A line is parallel to xy-plane if all the points on the line have equal ....

Sol. z-coordinates
Hence, the value of the filler is $\mathbf{z}$**-coordinates**.

Q44. A line is parallel to x-axis if all the points on the line have equal ....

Sol. y and z coordinates.
Hence, the value of the filler is $\mathbf{y}$ **and** $\mathbf{z}$ **coordinates**.

Q45. $x = a$ represents a plane parallel to ....

Sol. $x = a$ represents a plane parallel to $\mathbf{yz}$**-plane**.

Q46. The plane parallel to yz-plane is perpendicular to ....

Sol. The plane parallel to yz-plane is perpendicular to x-axis.
Hence, the value of the filler is $\mathbf{x}$**-axis**.

Q47. The length of the longest piece of a string that can be stretched straight in a rectangular room whose dimensions are 10, 13 and 8 units are ....

Sol. The given dimensions are 10, 13 and 8

Let $a = 10$, $b = 13$ and $c = 8$

$$\therefore \quad \text{Required length} = \sqrt{a^2 + b^2 + c^2}$$
$$= \sqrt{(10)^2 + (13)^2 + (8)^2}$$
$$= \sqrt{100 + 169 + 64} = \sqrt{333}$$

Hence, the value of the filler is $\mathbf{\sqrt{333}}$.

Q48. If the distance between the points $(a, 2, 1)$ and $(1, -1, 1)$ is 5, then $a =$ ....

Sol. Given points are $(a, 2, 1)$ and $(1, -1, 1)$

$$\therefore \quad \text{Distance} = \sqrt{(a-1)^2 + (2+1)^2 + (1-1)^2}$$
$$5 = \sqrt{a^2 + 1 - 2a + 9}$$

Squaring both sides, we have

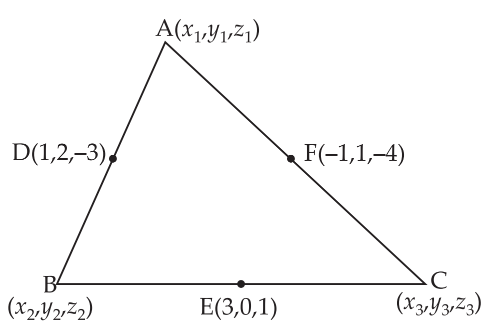
$$25 = a^2 - 2a + 10$$
$$\Rightarrow \quad a^2 - 2a - 15 = 0$$
$$\Rightarrow \quad a^2 - 5a + 3a - 15 = 0$$
$$\Rightarrow \quad a(a-5) + 3(a-5) = 0$$
$$\Rightarrow \quad (a+3)(a-5) = 0$$
$$\therefore \quad a = -3 \text{ or } 5$$

Hence, the value of the filler is **5 or – 3**.

Q49. If the mid-points of the sides of a triangle AB, BC and CA are D(1, 2, – 3), E(3, 0, 1) and F(– 1, 1, – 4), then the centroid of the ΔABC is ....

Sol. Let the vertices of the ΔABC be

A(x_1, y_1, z_1), B(x_2, y_2, z_2) and C(x_3, y_3, z_3).



D is the mid-point of AB

$\therefore \quad 1 = \dfrac{x_1 + x_2}{2} \Rightarrow x_1 + x_2 = 2 \quad ...(i)$

$2 = \dfrac{y_1 + y_2}{2} \Rightarrow y_1 + y_2 = 4 \quad ...(ii)$

$-3 = \dfrac{z_1 + z_2}{2} \Rightarrow z_1 + z_2 = -6 \quad ...(iii)$

E is the mid-point of BC

$\therefore \quad 3 = \dfrac{x_2 + x_3}{2} \Rightarrow x_2 + x_3 = 6 \quad ...(iv)$

$0 = \dfrac{y_2 + y_3}{2} \Rightarrow y_2 + y_3 = 0 \quad ...(v)$

$1 = \dfrac{z_2 + z_3}{2} \Rightarrow z_2 + z_3 = 2 \quad ...(vi)$

F is the mid-point of AC

$\therefore \quad -1 = \dfrac{x_1 + x_3}{2} \Rightarrow x_1 + x_3 = -2 \quad ...(vii)$

$1 = \dfrac{y_1 + y_3}{2} \Rightarrow y_1 + y_3 = 2 \quad ...(viii)$

$-4 = \dfrac{z_1 + z_3}{2} \Rightarrow z_1 + z_3 = -8 \quad ...(ix)$

Adding eq. (*i*), (*iv*) and (*vii*) we get

$2(x_1 + x_2 + x_3) = 2 + 6 - 2$

$\therefore \quad x_1 + x_2 + x_3 = 3 \quad ...(x)$

Adding (*ii*), (*v*) and (*viii*) we get

$2(y_1 + y_2 + y_3) = 4 + 0 + 2$

$\Rightarrow \quad y_1 + y_2 + y_3 = 3 \quad ...(xi)$

Adding (*iii*), (*vi*) and (*ix*) we get

$2(z_1 + z_2 + z_3) = -6 + 2 - 8$

$\therefore \quad z_1 + z_2 + z_3 = -6 \quad ...(xii)$

Subtracting (*i*) from eq. (*x*) we get

$x_3 = 3 - 2 = 1 \Rightarrow x_3 = 1$

Subtracting (*iv*) from eq. (*x*) we get

$x_1 = 3 - 6 = -3 \Rightarrow x_1 = -3$

Subtracting (*vii*) from eq. (*x*) we get

$x_2 = 3 - (-2) = 5 \Rightarrow x_2 = 5$

Subtracting (*ii*) from eq. (*xi*) we get

$$y_3 = 3 - 4 = -1 \quad \Rightarrow \quad y_3 = -1$$

Subtracting (*viii*) from eq. (*xi*) we get

$$y_2 = 3 - 2 = 1 \quad \Rightarrow \quad y_2 = 1$$

Subtracting (*v*) from eq. (*xi*) we get

$$y_1 = 3 - 0 = 3 \quad \Rightarrow \quad y_1 = 3$$

Subtracting (*iii*) from eq. (*xii*) we get

$$z_3 = -6 - (-6) = 0 \quad \Rightarrow \quad z_3 = 0$$

Subtracting (*vi*) from eq. (*xii*) we get

$$z_1 = -6 - 2 = -8 \quad \Rightarrow \quad z_1 = -8$$

Subtracting (*ix*) from eq. (*xii*) we get

$$z_2 = -6 - (-8) = 2 \quad \Rightarrow \quad z_2 = 2$$

So, the coordinates are A(– 3, 3, – 8), B(5, 1, 2) and C(1, – 1, 0)

$$\therefore \quad \text{Centroid G} = \left(\frac{-3+5+1}{3}, \frac{3+1-1}{3}, \frac{-8+2+0}{3}\right)$$

$$= (1, 1, -2)$$

Hence, the value of the filler is **(1, 1, – 2)**.

Q50. Match each item under the Column I to its correct answer given under Column II.

	Column I		**Column II**
(*a*)	In *xy*-plane	(*i*)	Ist octant
(*b*)	Point (2, 3, 4) lies in the	(*ii*)	*yz*-plane
(*c*)	Locus of the points having *x*-coordinate 0 is	(*iii*)	*z*-coordinate is zero
(*d*)	A line is parallel to *x*-axis if and only	(*iv*)	*z*-axis
(*e*)	If $x = 0$, $y = 0$ taken together will represent the	(*v*)	Parallel to *xy*-plane
(*f*)	$z = c$ represent the plane.	(*vi*)	If all the points on the line have equal *y* and *z*-coordinates
(*g*)	Planes $x = a$, $y = b$ represent the line	(*vii*)	From the point on the respective axes
(*h*)	Coordinates of a point are the distances from the origin to the feet of perpendiculars	(*viii*)	Parallel to *z*-axis

(*i*)	A ball is a solid region in the space enclosed by a	(*ix*)	disc
(*j*)	Region in the plane enclosed by a circle is known as a	(*x*)	sphere

Sol. (*a*) In xy-plane, z-coordinate is zero.
Hence, (*a*) $\leftrightarrow$ (*iii*).

(*b*) The point (2, 3, 4) lies in first octant.
Hence, (*b*) $\leftrightarrow$ (*i*).

(*c*) Locus of the points with x-coordinate is zero is yz-plane.
Hence, (*c*) $\leftrightarrow$ (*ii*).

(*d*) A line is parallel to x-axis if and only if all the points on the line have equal y and z-coordinates.
Hence, (*d*) $\leftrightarrow$ (*vi*).

(*e*) $x = 0$, $y = 0$ represent z-axis.
Hence, (*e*) $\leftrightarrow$ (*iv*).

(*f*) $z = c$ represent a plane parallel to xy-plane.
Hence, (*f*) $\leftrightarrow$ (*v*).

(*g*) The planes $x = a$, $y = b$ represent the line parallel to z-axis.
Hence, (*g*) $\leftrightarrow$ (*viii*).

(*h*) Coordinates of a point are the distances from the origin to the feet of perpendicular from the point on the respective axes.
Hence, (*h*) $\leftrightarrow$ (*vii*).

(*i*) A ball is solid region in the space enclosed by a sphere.
Hence, (*i*) $\leftrightarrow$ (*x*).

(*j*) The region in the plane enclosed by a circle is known as a disc.
Hence, (*j*) $\leftrightarrow$ (*ix*).