2. POLYNOMIALS

POLYNOMIAL

An algebraic expression in one variable x, of the form $p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_2x^2 + a_1x + a_0$, where $a_0, a_1, a_2, ..., a_n$ are real numbers (constants), $a_n \neq 0$ and all the exponents of x are non-negative integers is called a polynomial in x. a_nx^n, $a_{n-1}x^{n-1}$, $a_{n-2}x^{n-2}$ .... a_2x^2, a_1x, a_0 are known as terms of polynomials and $a_0, a_1, a_2, ..., a_n$ are also known as coefficients of polynomials.

Example : $f(x) = 4x + 3$, $g(x) = 3x^2 + 9x - 3$, $p(x) = \frac{1}{2}x^3 + \frac{3}{4}x^2 - x + 3$

PRACTICE PROBLEMS

1. Which of the following expressions are polynomials in one variable and which are not? State reasons

(i) $x^2 + 2x - 5$ (ii) $5t^3 - 3t^5 - 5\sqrt{2}$ (iii) $4s + \frac{1}{s}$ (iv) $\frac{(x + x^2)}{x}$ (v) $x + \sqrt{7}x^3 + x^2$ (vi) $\sqrt{r} + \frac{1}{\sqrt{r}}$

DEGREE OF A POLYNOMIAL

The exponent of the highest degree term in a polynomial is known as the degree of the polynomial.

Example : (i) $f(x) = 4x + 5$ is a polynomial of degree 1. (ii) $g(x) = 5x^2 + 2x - 5$ is a polynomial of degree 2.

ILLUSTRATION

Q.1 Find out the degree of following polynomials.

(i) $p(x) = 7x + 5x^2 - \sqrt{3}$ (ii) $q(t) = 5t^4 - 32t^2 + 5t - 8$ (iii) $r(p) = p^3 - p^6 - 5\sqrt{2}$ (iv) $h(x) = \frac{1}{2} - 3x$

Sol. (i) p(x) is of degree 2 as highest powered term is $5x^2$. (ii) q(t) is of degree 4.
(iii) r(p) is of degree 6. (iv) h(x) is of degree 1.

PRACTICE PROBLEMS

2. Write the degree of polynomial p(x) gives as:

(i) $p(x) = x^2 + \frac{3}{2}x + 1$ (ii) $p(x) = 3x^3 - \frac{7}{2}x + \sqrt{3}$ (iii) $p(x) = \sqrt{7}x^4 + 2x^3 + \sqrt{9}x + 4$

(iv) $p(x) = \frac{3}{4}x - 5$ (v) $p(x) = -4x^3 - \frac{1}{\sqrt{3}}x^2 + x^4$ (vi) $p(x) = -\sqrt{5}$

TYPES OF POLYNOMIALS

0. Constant Polynomial: A polynomial of degree zero is called a constant polynomial.

Example : $f(x) = 5$, $q(x) = \frac{5}{2}$, $r(x) = -\frac{7}{5}$

The constant polynomials 0 (zero) is known as the zero polynomial. The degree of zero polynomial is ***not defined*** because $f(x) = 0$, $g(x) = 0x^4$, $h(x) = 0x^6$, $p(x) = 0x^{12}$ are all equal to zero polynomials.

1. Linear polynomial: A polynomial of degree 1 is called a linear polynomial.

Example : $f(x) = 7x$, $q(y) = \frac{4}{3}y + 8$, $r(t) = -\frac{4}{7}t - 7$, $h(x) = \sqrt{3}x + 7$

2. **Quadratic polynomial:** A polynomial of degree 2 is called a quadratic polynomial.

Example : $f(y) = 4y^2$, $q(s) = \frac{2}{5}s^2 + 8$, $r(x) = \sqrt{3}x^2 + 7x + 9$

3. **Cubic polynomial:** A polynomial of degree 3 is called a cubic polynomial.

Example : $f(t) = 7t^3$, $p(r) = \frac{2}{5}r^3 + 9r^2 + 3r + 8$, $h(x) = \sqrt{5}x^2 + 7x + 9x^3 + 2$

4. **Bi-Quadratic polynomial:** A polynomial of degree 4 is called a bi-quadratic polynomial.

Example : $p(x) = 3x^4 + 5x^3 + 2x^2 - 6x - 3$, $q(t) = 5t^4 + 9t^2 + 4$

PRACTICE PROBLEMS

3. Find the types of the polynomials on the basis of degree:

(i) $\sqrt{5}x^2 + 6x + 3$ (ii) $x^4 + 2x^2 + 2$ (iii) 5 (iv) 4y + 8 (v) $12s^3$

VALUE OF A POLYNOMIAL

Value of a polynomial f(x) at x = α is obtained by substituting x = α in the given polynomial and is denoted by f(α).

ILLUSTRATION

Q.2 Find the value of $p(x) = 3x^2 + 5x - 4$ at x = – 2 and x = 5

Sol. $p(-2) = 3(-2)^2 + 5(-2) - 4 = 3(4) - 10 - 4 = 12 - 14 = -2$ **Ans**

$p(5) = 3(5)^2 + 5(5) - 4 = 3(25) + 25 - 4 = 75 + 21 = 96$ **Ans**

ZERO OF A POLYNOMIAL

A real number c is said to be a zero of a polynomial p(x), if p(c) = 0. The zeroes of polynomial p(x) are actually the x-coordinates of the points where the graph of y = p(x) intersects the x-axis.

Example : Let p(x) = 4x – 8, if we put x = 2, then p(2) = 4 (2) – 8 = 8 – 8 = 0, so 2 is a zero of p(x)

POINT TO NOTE:

* A linear polynomial can have at most one zero.
* A quadratic or cubic polynomial can have at most two and three zeroes respectively.
* In general, a polynomial of degree n has atmost n zeroes.
* A polynomial can have minimum 0 (zero) zeroes.

ILLUSTRATION

Q.3 Find the zeroes of the polynomial $p(x) = x^2 - 10x - 75$

Sol. We have, $p(x) = x^2 - 10x - 75 = x^2 - 15x + 5x - 75 = x(x - 15) + 5(x - 15) = (x - 15)(x + 5)$

$\therefore$ p(x) = (x – 15) (x + 5) So, p(x) = 0 when x = 15 or x = –5. Therefore required zeroes are 15 and –5.

Q.4 Find the zeroes of the polynomial $3x^2 - x - 4$.

Sol. $P(x) = 3x^2 - x - 4 = 3x^2 - 4x + 3x - 4 = x(3x - 4) + 1(3x - 4) = (x + 1)(3x - 4)$

zeroes of the polynomial, P(x) = 0 So, $(x + 1)(3x - 4) = 0 \Rightarrow x + 1 = 0, 3x - 4 = 0$

$\Rightarrow x = -1$, $x = \frac{4}{3}$ are the zeroes of the polynomial P(x)

Q.5 Show that 2 is not a zero of the polynomial, $P(x) = x^2 + 2x + 5$

Sol. $P(x) = x^2 + 2x + 5$, then $P(2) = (2)^2 + 2(2) + 5 = 4 + 4 + 5 = 13 \neq 0$

since, $P(2) \neq 0$, 2 is not a zero of the polynomial $P(x)$.

PRACTICE PROBLEMS

4. Find the zeroes of the following polynomials:

(i) $x^2 - x - 6$ (ii) $3y^2 - 12$ (iii) $5t^2 + 30t$ (iv) $9x^2 + 3x - 2$

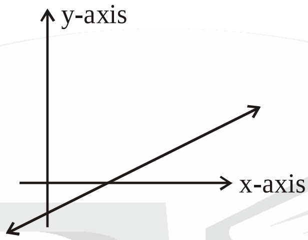
(v) $(2x + 5)^2$ (vi) $8 - 4\sqrt{2}x + x^2$ (vii) $4x^2$ (viii) $(x - 3)(x + 4)$

GRAPH OF POLYNOMIAL

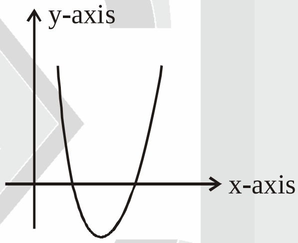
Geometric Meaning of the zeroes of a polynomial

In algebraic language, the graph of a polynomial $f(x)$ is the collection of all points (x, y) where $y = p(x)$

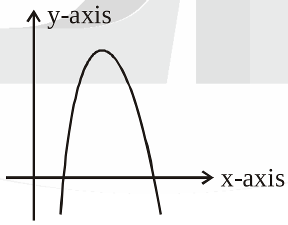
(i) Graph of a linear polynomial $p(x) = ax + b$ is a straight line.



(ii) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola open upwards like $\cup$ if $a > 0$.



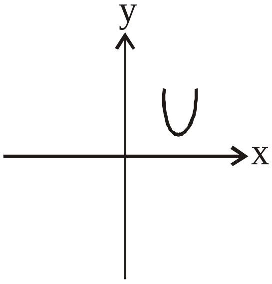
(iii) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola open downwards like $\cap$ if $a < 0$.



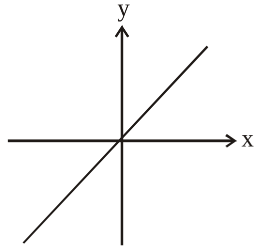
(iv) In general a polynomial $p(x)$ of degree n crosses the x-axis at atmost n points.

Zeroes of a polynomial with respect to graph of the polynomial:

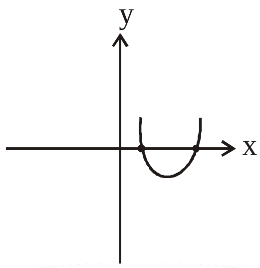
(i) When the graph of quadratic polynomial does not cut the x-axis at any point. The quadratic polynomial $ax^2 + bx + c$ has no zero.



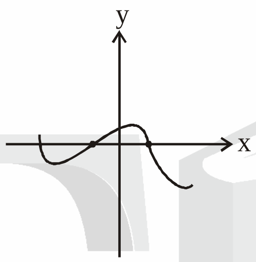
(ii) When the graph cut x-axis at exactly one point. There is only one zero for the quadratic polynomial ax^2+bx+c



(iii) When the graph cut x-axis at two distinct points there are two zeroes of quadratic polynomial $ax^2 + bx + c$

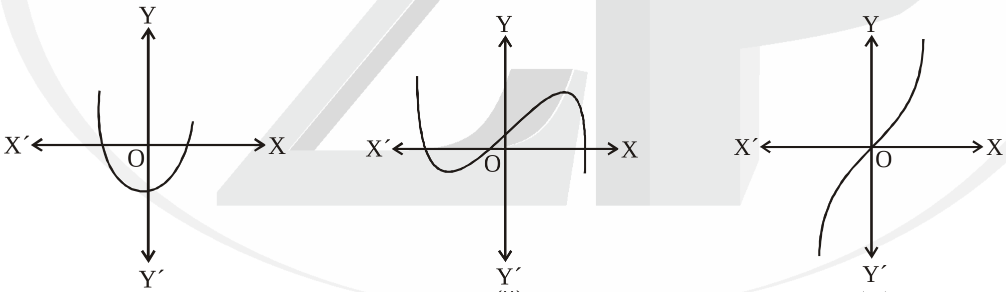


(iv) When the graph cut x-axis at three distinct points there are three zeroes of cubic polynomial ax^3+bx^2+cx+d



ILLUSTRATION

Q.6 The graphs of $y = p(x)$ are given below. Find the number of zeroes of $p(x)$ in each case:



(i) (ii) (iii)

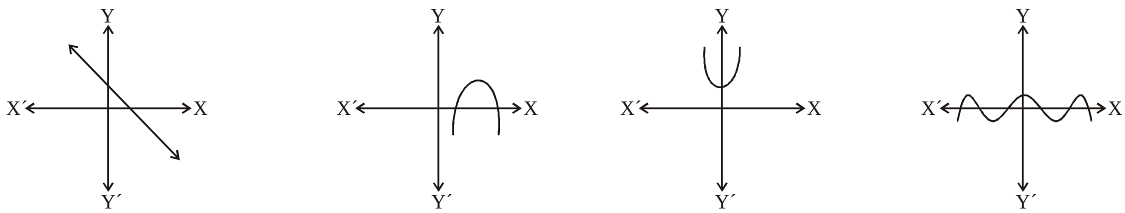
Sol. (i) The number of zeroes is 2 as the given curve intersects x-axis at two points.

(ii) The given curve intersects x-axis at three points so, the number of zeroes is 3.

(iii) The curve intersects only at one point, therefore, required number of zeroes is one.

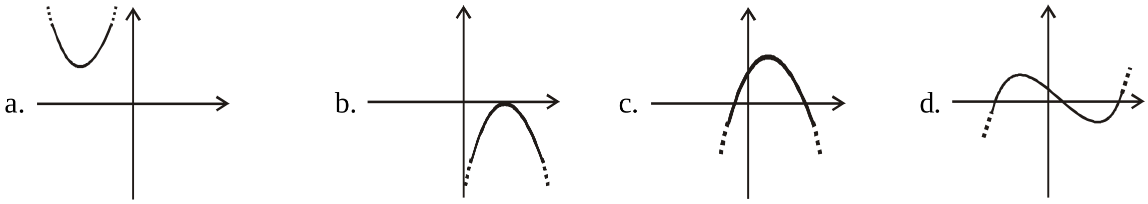
PRACTICE PROBLEMS

5. Each of the graph of $y = P(x)$, where $P(x)$ is a polynomial for each of the graph, find the number of the zeroes of $P(x)$.

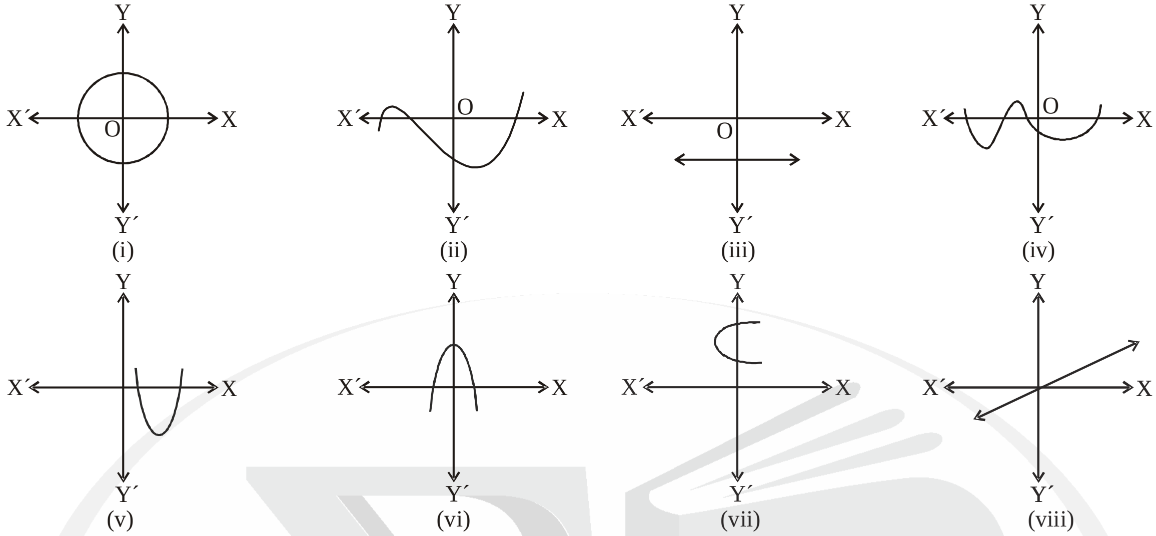


(i) (ii) (iii) (iv)

6. Which of the following is not the graph of a quadratic polynomial?



7. The graph of y = p(x) given below. Find the number of zeroes of p(x), in each case:–



RELATIONSHIP BETWEEN ZEROES AND COEFFICIENTS OF A POLYNOMIAL

1. If α, β are the zeroes of quadratic polynomial $p(x) = ax^2 + bx + c$, $a \neq 0$ then, sum of zeroes $= \alpha + \beta = -\frac{b}{a}$,

product of zeroes $= \alpha\beta = \frac{c}{a}$

2. If α, β and γ are the zeroes of cubic polynomial, $p(x) = ax^3 + b^2 + cx + d$, $a \neq 0$, then,

sum of zeroes $= \alpha + \beta + \gamma = -\frac{b}{a}$, Sum of product of zeroes $= \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$, product of zeroes $= \alpha\beta\gamma = -\frac{d}{a}$

ILLUSTRATION

Q.7 Find the zeroes of the polynomial $3x^2 - 10x + 8$ and verify the relationship between the zeroes and the coefficients.

Sol. Let us factorise $3x^2 - 10x + 8$ and find its two zeroes

$3x^2 - 10x + 8 = 3x^2 - 6x - 4x + 8 = 3x(x - 2) - 4(x - 2) = (3x - 4)(x - 2)$

So, $3x - 4 = 0 \Rightarrow x = \frac{4}{3}$ and $x - 2 = 0 \Rightarrow x = 2$; Let $\alpha = \frac{4}{3}$ and $\beta = 2$

Direct Method: $\alpha + \beta = \frac{4}{3} + 2 = \frac{4+6}{3} = \frac{10}{3}$ and $\alpha \times \beta = \frac{4}{3} \times 2 = \frac{8}{3}$

Formula Method: $3x^2 - 10x + 8$, Let $a = 3$, $b = -10$, $c = 8$

Sum of zeroes $= \alpha + \beta = \frac{-b}{a} = \frac{-(-10)}{3} = \frac{10}{3}$ and Product of zeroes $= \alpha \times \beta = \frac{c}{a} = \frac{8}{3}$

Q.8 Find the zeroes of the polynomial $5x^2 - 15x$ and verify the relationship between the zeroes and the coefficients.

Sol. Let us factorise $5x^2 - 15x$ by taking out common factors and find its two zeroes

$5x^2 - 15x = 5x(x - 3)$; So, $5x = 0 \Rightarrow x = \frac{0}{5} = 0$ and $x - 3 = 0 \Rightarrow x = 3$

Let $\alpha = 0$ and $\beta = 2$; Direct Method: $\alpha + \beta = 0 + 3 = 3$ and $\alpha \times \beta = 0 \times 3 = 0$

Formula Method: $5x^2 - 15x$, Let $a = 5$, $b = -15$, $c = 0$

Sum of zeroes $= \alpha + \beta = \frac{-b}{a} = \frac{-(-15)}{5} = 3$ and Product of zeroes $= \alpha \times \beta = \frac{c}{a} = \frac{0}{5} = 0$

Q.9 Find the zeroes of the polynomial $x^2 - 6$ and verify the relationship between the zeroes and the coefficients.

Sol. Let us factorise $x^2 - 6$ by using identity $a^2 - b^2 = (a + b)(a - b)$ and find its two zeroes

$x^2 - 6 = x^2 - (\sqrt{6})^2 = (x + \sqrt{6})(x - \sqrt{6})$

So, $x + \sqrt{6} = 0 \Rightarrow x = -\sqrt{6}$ and $x - \sqrt{6} = 0 \Rightarrow x = \sqrt{6}$

Let $\alpha = -\sqrt{6}$ and $\beta = \sqrt{6}$

Direct Method: $\alpha + \beta = -\sqrt{6} + \sqrt{6} = 0$ and $\alpha \times \beta = -\sqrt{6} \times \sqrt{6} = -6$

Formula Method: $x^2 - 6$, Let $a = 1$, $b = 0$, $c = -6$

Sum of zeroes $= \alpha + \beta = \frac{-b}{a} = \frac{0}{1} = 0$ and Product of zeroes $= \alpha \times \beta = \frac{c}{a} = \frac{-6}{1} = -6$

Q.10 If α and β are the zeros of the quadratic polynomial $f(x) = x^2 - px + q$, then find the values of

(i) $\alpha^2 + \beta^2$ (ii) $\frac{1}{\alpha} + \frac{1}{\beta}$

Sol. Since α and β are the zero of the polynomial $f(x) = x^2 - px + q$.

$\therefore \alpha + \beta = -\left(\frac{-p}{1}\right) = p$ and $\alpha\beta = \frac{q}{1} = q$

(i) We have, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$\Rightarrow \alpha^2 + \beta^2 = p^2 - 2p$ [$\because \alpha + \beta = p$ and $\alpha\beta = q$]

(ii) We have, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{p}{q}$

Q.11 If α and β are the zeros of the quadratic polynomial $f(x) = ax^2 - bx + c$, then evaluate:

(i) $\alpha^2 + \beta^2$ (ii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ (iii) $\alpha^3 + \beta^3$ (iv) $\frac{1}{\alpha^3} + \frac{1}{\beta^3}$ (v) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

Sol. Since α and β are the zeros of the quadratic polynomial $f(x) = ax^2 + bx + c$.

$\therefore \alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

(i) We have, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$\Rightarrow \alpha^2 + \beta^2 = \left(\frac{-b}{a}\right)^2 - \frac{2c}{a} = \frac{b^2 - 2ac}{a^2}$

(ii) We have, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{\left(\frac{-b}{a}\right)^2 - 2\left(\frac{c}{a}\right)}{\frac{c}{a}}$

$$\Rightarrow \quad \frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{b^2-2ac}{ac}$$

(iii) We have, $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$$\Rightarrow \quad \alpha^3+\beta^3 = \left(\frac{-b}{a}\right)^3 - 3\frac{c}{a}\left(\frac{-b}{a}\right) = \frac{-b^3}{a^3}+\frac{3bc}{a^2}=\frac{-b^3+3abc}{a^3}=\frac{3abc-b^3}{a^3}$$

(iv) We have, $\frac{1}{\alpha^3}+\frac{1}{\beta^3}=\frac{\alpha^3+\beta^3}{(\alpha\beta)^3}=\frac{\frac{3abc-b^3}{a^3}}{\left(\frac{c}{a}\right)^3}$ [Using (iv)]

$$\Rightarrow \quad \frac{1}{\alpha^3}+\frac{1}{\beta^3}=\frac{3abc-b^3}{c^3}$$

(v) We have, $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}=\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}=\frac{\left(-\frac{b}{a}\right)^3-3\left(\frac{c}{a}\right)\left(-\frac{b}{a}\right)}{\frac{c}{a}} = \frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{3abc-b^3}{a^2c}$

Q.12 If α and β are the zeros of the quadratic polynomial $f(x) = ax^2 + bx + c$, then evaluate:

(i) $\alpha^4 + \beta^4$ (ii) $\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}$

Sol. Since and are the zeros of the quadratic polynomial $f(x) = ax^2 + bx + c$.

$\therefore\ \alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

(i) We have, $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$

$$\Rightarrow \quad \alpha^4 + \beta^4 = \{(\alpha + \beta)^2 - 2\alpha\beta\}^2 - 2(\alpha\beta)^2$$

$$\Rightarrow \quad \alpha^4+\beta^4 = \left\{\left(-\frac{b}{a}\right)^2-2\frac{c}{a}\right\}^2-2\left(\frac{c}{a}\right)^2 \qquad \left[\because \alpha+\beta=-\frac{b}{a}, \alpha\beta=\frac{c}{a}\right]$$

$$\Rightarrow \quad \alpha^4+\beta^4 = \left(\frac{b^2-2ac}{a^2}\right)^2-\frac{2c^2}{a^2}$$

$$\Rightarrow \quad \alpha^4+\beta^4 = \frac{(b^2-2ac)^2-2a^2c^2}{a^4}$$

(ii) We have, $\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}=\frac{\alpha^4+\beta^4}{\alpha^2\beta^2}=\frac{(b^2-2ac)^2-2a^2c^2}{a^4\times\left(\frac{c}{a}\right)^2}$ [Using (i)]

$$\Rightarrow \quad \frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}=\frac{(b^2-2ac)^2-2a^2c^2}{a^2c^2}$$

PRACTICE PROBLEMS

8. Find the zeroes of the quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) $x^2 + 5x - 36$ (ii) $9x^2 + 3x - 2$ (iii) $6x^2 - 3 - 7x$ (iv) $x^2 + 4x$

(v) $3x^2 + 6x$ (vi) $5x^2 - 9x$ (vii) $x^2 - 9$ (viii) $3x^2 - 5$

9. If α and β are the zeros of the quadratic polynomial $f(x) = 5x^2 + 8x + 3$, then evaluate:

(i) $\alpha^2 + \beta^2$ (ii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ (iii) $\alpha^3 + \beta^3$ (iv) $\frac{1}{\alpha^3} + \frac{1}{\beta^3}$ (v) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

FORMATION OF QUADRATIC AND CUBIC POLYNOMIALS

If α, β are zeroes of a quadratic polynomial p(x), then k { $x^2 - (\alpha + \beta) x + \alpha\beta$ } is the quadratic polynomial or k { x^2 – (Sum of roots) x + Product of roots } where k is a real number.

If α, β, γ are zeroes of a cubic polynomial p(x), then k { $x^3 - (\alpha + \beta + \gamma) x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha) x + \alpha\beta\gamma$ } is the cubic polynomial or k { x^3 – (Sum of roots) x^2 + (Sum of product of roots) x + Product of roots }

ILLUSTRATION

Q.13 Write a quadratic polynomial, the sum and product of whose zeroes are –7 and 10 respectively.

Sol. Let α, β be zeroes then, $\alpha + \beta = -7$, $\alpha\beta = 10$.

So, required polynomial p(x) is given by

$= x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - (-7)x + 10 \quad \therefore \quad p(x) = x^2 + 7x + 10$

Q.14 Write a quadratic polynomial, whose zeroes are $\frac{2}{3}$ and 5.

Sol. Let $\alpha = \frac{2}{3}, \beta = 5$, $\alpha + \beta = \frac{2}{3} + 5 = \frac{2+15}{3} = \frac{17}{3}$, $\alpha \times \beta = \frac{2}{3} \times 5 = \frac{10}{3}$

So, required polynomial p(x) is given by $= x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - \frac{17}{3}x + \frac{10}{3}, \frac{1}{3}(3x^2 - 17x + 10)$

Q.15 Find the polynomial having $5 \pm \sqrt{3}$ as its zeroes.

Sol. Let $\alpha = 5+\sqrt{3}$, $\beta = 5-\sqrt{3}$ $\alpha + \beta = 5 + \sqrt{3} + 5 - \sqrt{3} = 10$,

$\alpha \times \beta = (5 + \sqrt{3}) + (5 - \sqrt{3}) = 5^2 - (\sqrt{3})^2 = 25 - 3 = 22$

$= x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - (10)x + 22 \therefore p(x) = x^2 - 10x + 22$

PRACTICE PROBLEMS

10. Find a quadratic polynomial, the sum and the product of whose zeroes are –5 & $\frac{1}{2}$ respectively.

11. Find a quadratic polynomial whose zeroes are 8 and 10.

12. Find a quadratic each with the given numbers as the sum and product of its zeroes respectively.

(i) 5, –2 (ii) a–b, –ab (iii) –4, –21 (iv) –3, –3 (v) $3-\sqrt{2}, -3\sqrt{2}$

13. Form quadratic polynomial each with given pair of zeroes as

(i) $\left(5, -\frac{1}{5}\right)$ (ii) $2+3\sqrt{5}, 2-3\sqrt{5}$ (iii) $\left(\frac{2a+b}{3}, \frac{a-2b}{3}\right)$ (iv) $p^2 + q^2, p^2 - q^2$

• DIVISION ALGORITHM FOR POLYNOMIALS •

1. If p(x) and q(x) are any two polynomials then we always have polynomials q(x) and r(x) such that p(x) = g(x). q(x) + r(x) where $g(x) \neq 0$ and r(x) = 0 or degree of r(x) < degree of g(x).
2. In particular, if r(x) = 0, then g(x) is a divisor of p(x) so g(x) is a factor of p(x).

ILLUSTRATION

Q.16 Divide p (x) = $x^2 + 3x^3 + 2x + 5$ (cubic polynomial) by g(x) = $1 + 2x + x^2$ (quadratic polynomial)

Sol.

$$
\begin{array}{r|l}
 & 3x - 5 \\
x^2 + 2x + 1 & 3x^3 + x^2 + 2x + 5 \\
 & 3x^3 + 6x^2 + 3x \\
 & - \quad - \quad - \\
\hline
 & -5x^2 - x + 5 \\
 & -5x^2 - 10x - 5 \\
 & + \quad + \quad + \\
\hline
 & 9x + 10
\end{array}
$$

Verification

Now, Divisor × Quotient + Remainder

$= (x^2 + 2x + 1)(3x - 5) + (9x + 10) = 3x^3 - 5x^2 + 6x^2 - 10x + 3x - 5 + 9x + 10 = 3x^3 + x^2 + 2x + 5$ = Dividend

Thus, the division algorithm is verified.

PRACTICE PROBLEMS

14. Divide $3x^3 - 5x^4 + 5x^2 + 6x - 7$ by $x + 1 - x^2$ and verify the division algorithm.
15. Divide $3x^2 + 5x + 11$ by x + 3 and verify the division algorithm.
16. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial with the first polynomial: $x^2 - 2x + 1$, $x^4 - 2x^3 + 2x^2 - 2x + 1$.

PRACTICE PROBLEMS ANSWERS

1. (i) Polynomial (ii) Polynomial (iii) Not a polynomial (iv) Not a polynomial (v) Polynomial (vi) Not a polynomial

2. (i) 2 (ii) 3 (iii) 4 (iv) 1 (v) 4 (vi) 0

3. (i) Quadratic (ii) Bi-Quadratic (iii) Constant (iv) Linear (v) Cubic

4. (i) 3, –2 (ii) –2, 2 (iii) 0, –6 (iv) –2/3, 1/3 (v) –5/2, –5/2 (vi) $2\sqrt{2}$, $2\sqrt{2}$ (vii) 0, 0 (viii) 3,–43

5. (i) 2 (ii) 2 (iii) 0 (iv) 6

6. Option (d) is not a quadratic polynomial

7. (i) 2 (ii) 3 (iii) 0 (iv) 4 (v) 2 (vi) 2 (vii) 0 (viii) 1

8. (i) –9, 4 (ii) –2/3, 1/3 (iii) 3/2, –1/3 (iv) 0, –4 (v) 0, –2 (vi) 0, 9/5 (vii) 3, –3 (viii) $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}$

9. (i) $\frac{34}{25}$ (ii) $\frac{34}{15}$ (iii) $\frac{19}{25}$ (iv) $\frac{95}{27}$ (v) $\frac{19}{15}$

10. $x^2 - 5x + 1/2$ or $2x^2 - 10x + 1$ **11.** $x^2 - 18x + 80$ **12.** (i) $P(x) = x^2 - 5x - 2$

(ii) $P(x) = x^2 - (a - b)x - ab$ (iii) $P(x) = x^2 + 4x - 21$ (iv) $P(x) = x^2 + 3x - 3$ (v) $P(x) = x2 - \left(3 - \sqrt{2}\right)x - 3\sqrt{2}$

13. (i) $P(x) = 5x^2 - 24x - 5$ (ii) $P(x) = x^2 - 4x - 41$ (iii) $P(x) = 9x^2 - 3(3a - b)x + (2a^2 - 3ab - 2b^2)$

(iv) $P(x) = x^2 - 2P^2x + p^4 + q^4$ **14.** $q(x) = -5x^2 - 8x - 18$; $r(x) = 32x + 11$

15. q(x) = 3x – 4 ; r(x) = 23 **16.** Yes, $q(x) = x^2 + 1$; r(x) = 0.

X MATHS POLYNOMIALS (EXCERCISE)

A. ZEROES OF POLYNOMIAL & VERIFY RELATIONSHIP BETWEEN ZEROES & COEFFICIENT

Find zeroes of following polynomials & hence verify the relationship between zeroes and coefficient of polynomials: (Q. 1 - Q. 19)

1. $P(x) = x^2 + 5x + 6$ **2.** $P(x) = x^2 - 7x + 10$ **3.** $P(x) = x^2 + x - 6$ **4.** $P(x) = 6x^2 - 7x - 3$

5. $P(x) = 3x^2 - 17x - 6$ **6.** $P(x) = 4x^2 - 4x + 1$ **7.** $P(x) = x^2 - 3$ **8.** $P(x) = x^2 - 5$

9. $P(x) = 8x^2 - 4$ **10.** $P(u) = 5u^2 + 10u$ **11.** $P(x) = 9x^2 - 3x$ **12.** $P(y) = 12y^2 - 5y$

13. $P(x) = \sqrt{3}x^2 + 10x + 7\sqrt{3}$ **14.** $P(x) = x^2 - (\sqrt{3}+1)x + \sqrt{3}$ **15.** $P(x) = 4\sqrt{3}x^2 + 5x - 2\sqrt{3}$

16. $P(x) = a(x^2 + 1) - x(a^2 + 1)$ **17.** $P(x) = abx^2 + (b^2 - ac)x - bc$

18*. $P(x) = 2x^3 + x^2 - 5x + 2$ **19*.** $P(x) = x^3 - 4x^2 + 5x - 2$

20*. Verify that 3, –2, 1 are the zeroes of the cubic polynomial $p(x) = x^3 - 2x^2 - 5x + 6$ and verify the relation between its zeroes and coefficients.

21*. Verify that 5, –2 and 1/3 are the zeroes of the cubic polynomial $p(x) = 3x^3 - 10x^2 - 27x + 10$ and verify the relation between its zeroes and coefficients.

B. FORMATION OF QUADRATIC POLYNOMIALS

Form quadratic polynomials if zeroes are given as:[USE:k(x^2–(sum) x + (product)]

22. 3 and 2 **23.** 3 and 1/9 **24.** –5 and –6 **25.** $\sqrt{3}$ and $\sqrt{5}$

26. $\sqrt{7}$ and $-\sqrt{7}$ **27.** $\frac{\sqrt{2}}{\sqrt{3}}$ and $-\frac{\sqrt{2}}{\sqrt{3}}$ **28.** $\sqrt{3}$ and $2\sqrt{3}$ **29.** $\frac{\sqrt{3}}{4}$ and $-\frac{3}{\sqrt{2}}$

30. $3+\sqrt{5}$ & $3-\sqrt{5}$ **31.** $3+\sqrt{2}$ and $3-\sqrt{2}$

FORM QUADRATIC POLYNOMIALS IF SUM OF ZEROES & PRODUCT OF ZEROS ARE GIVEN AS:

32. a. –5 and 6 **b.** 5/2 and 1 **33.** 0 and – 9 **34.** 2/3 and –5

C. FORMATION OF CUBIC POLYNOMIALS

***Form cubical poly. if zeroes are given as :**

35. 3, ½, –1 **36.** –2, –3, –1 **37.** 1, ½, –2 **38.** –5, 2, –14

***Form cubical polynomial if sum, sum of product when two are taken at a time and product of zeroes are given :**

39. 1, –10, 8 **40.** 4, 1, –6 **41.** 0, –19, –30 **42.** 1, ½, –2

D. DIVISION ALGORITHM & ITS APPLICATION

Divide *f(x)* by *g(x)* and find quotient & remainder, hence verify division algorithm.

43. $f(x) = 2x^2 + x - 15$ and $g(x) = x + 3$ **44.** $f(x) = 16 - 17x - 5x^2$ and $g(x) = 3 - 5x$.

45. $f(x) = 3x^3 - 4x^2 + 7x - 2$ and $g(x) = 2 - x + x^2$ **46.** $f(x) = 16 + 19x + x^2 - 6x^3$ & $g(x) = 2 + 5x - 3x^2$

Check whether the first polynomial is a factor of second polynomial by applying the division algorithm : (Q. 47 to Q. 48)

47. $g(t) = t^2 - 3$, $f(t) = 2t^4 + 3t^3 - 2t^2 - 9t - 12$ **48.** $g(x) = x^3 - 3x + 1$, $f(x) = x^5 - 4x^3 + x^2 + 3x + 1$

49. On dividing $3x^3 + 4x^2 + 5x - 13$ by g(x), the quotient and remainder are (3x+10) and (16x-43) respectively. Find g(x).

50. Divide $2x^4 - 9x^3 + 5x^2 + 3x - 8$ by $x^2 - 4x + 1$ and verify the division algorithm.

51. On dividing $5x^4 - 4x^3 + 3x^2 - 2x + 1$ by $x^2 + 2$, if the quotient is $ax^2 + bx + c$, find a, b, c.

52. Divide $6x^3 + 13x^2 + x - 2$ by 2x + 1, find the quotient and remainder.

53. Divide $30x^4 + 11x^3 - 82x^2 - 12x + 48$ by $3x^2 + 2x - 4$ & verify division algorithm.

54. On dividing the polynomial p(x) by a polynomial g(x) = $4x^2 + 3x - 2$, the quotient q(x)= $2x^2 + x - 1$ and remainder r(x) =14x – 10. Find the p(x).

55. Can (x+3) be the remainder on the division of a polynomial p(x) by (2x – 5)? Justify your answer?

E. Zeroes of a cubic or biquadratic polynomial given its one or two zeroes:

56. It being given that 1 is one of the zeroes of the polynomial $f(x) = 7x - x^3 - 6$. Find its other two zeroes.

57. Polynomial $P(x) = x^3 + 13x^2 + 32x + 20$, has – 2 as one zero, then find other two zeroes.

58. Polynomial $p(x) = 3x^3 - 5x^2 - 11x - 3$, has –1 as one zero, then find other two zeroes.

59. If – 2 and – 1 are two zeroes of the polynomial $P(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$, then find the other two zeroes.

60. If 1 and 3 are the zeroes of polynomial, $P(x) = x^4 - x^3 - 19x^2 + 49x - 30$, then find the other two zeroes.

61. If 1 and 2 are two zeroes of polynomial $p(x) = 2x^4 - 6x^3 + 3x^2 + 3x - 2$, then find the other two zeroes.

62. Obtain all zeroes of quadratic polynomial p(x) if its two zeroes are given below:

a. $p(x) = 2x^4 - 3x^3 - 3x^2 + 6x - 2$,
zeroes: $\sqrt{2}, -\sqrt{2}$

b. $p(x) = 2x^4 - 3x^3 - 5x^2 + 9x - 3$,
zeroes: $\sqrt{3}, -\sqrt{3}$

c. $p(x) = 2x^4 - 6x^3 + 3x^2 + 3x - 2$,
zeroes: $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$

d. $p(x) = x^4 - 7x^3 + 10x^2 + 14x - 24$,
zeroes: $\sqrt{2}, -\sqrt{2}$

e. $p(x) = 5x^4 - 5x^3 - 33x^2 + 3x + 18$,
zeroes: $\sqrt{\frac{3}{5}}, -\sqrt{\frac{3}{5}}$

f. $p(x) = x^4 - 3x^3 - x^2 + 9x - 6$,
zeroes: $\sqrt{3}, -\sqrt{3}$

g. $p(x) = 2x^4 - 10x^3 + 5x^2 + 15x - 12$,
zeroes: $\sqrt{\frac{3}{2}}, -\sqrt{\frac{3}{2}}$

h. $p(x) = x^4 + x^3 - 9x^2 - 3x + 18$,
zeroes: $\sqrt{3}, -\sqrt{3}$

i. $p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$,
zeroes: $2 \pm \sqrt{3}$

63. Two zeroes of polynomials are such that their sum is zero and the product is –6. Find its all zeores if

$f(x) = x^4 + x^3 - 12x^2 - 6x + 36$.

F. Miscellaneous Problems (Polynomials)

64. If α and β are the zeroes of polynomial $x^2 - (k + 6)x + 2(2k - 1)$, then find the value of k if $\alpha+\beta=\frac{1}{2}\alpha.\beta$.

65. If α and β are zeroes of the polynomials such that $\alpha + \beta = 24$ and $\alpha - \beta = 8$, find the quadratic polynomials.

66. If α and β are the zeroes of polynomial $x^2 - 6x + a$, then find the value of 'a' if $3\alpha + 2\beta = 20$

67. If α and β are the zeroes of polynomial $x^2 - 5x + 5$, then find the value of $\alpha^{-1}+\beta^{-1}$.

68. If one solution of the quadratic polynomial $3x^2 - 8x + 2k + 1$ is seven times the other. Find the solutions & the value of k.

69. If α and β are the zeroes of the quadratic polynomial $f(x)=x^2-x-4$, find the value of $\frac{1}{\alpha}+\frac{1}{\beta}-\alpha\beta$.

70. If α and β are the zeroes of polynomial $3x^2 + 5x - 2$, then form a quadratic polynomial whose zeroes are 2α and 2β.

71. If α and β are the zeroes of polynomial $x^2 - 2x + 5$, then form a quadratic polynomial whose zeroes are $\alpha+\beta$ and $\frac{1}{\alpha}+\frac{1}{\beta}$

72. If α and β are the zeroes of polynomial $x^2 - 2x - 8$, then form a quadratic polynomial whose zeroes are 3α and 3β.

73. If α and β are the zeroes of polynomial $x^2 - 3x + 7$, then form a quadratic polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$

74. If α and β are the zeroes of polynomial $25p^2 - 15p + 2$, then form a quadratic polynomial whose zeroes are $\frac{1}{2\alpha}$ and $\frac{1}{2\beta}$

75. If α and β are the zeroes of polynomial $21x^2 - x - 2$, then form a quadratic polynomial whose zeroes are 2α and 2β.

76. If α and β are the zeroes of the quadratic polynomial $f(x) = 6x^2 + x - 2$, find the value of $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$.

77. What must be added to the polynomial $p(x)=5x^4+6x^3-13x^2-44x+7$ so that the resulting polynomials is exactly divisible by the polynomial $q(x) = x^2+4x+3$ and the degree of the polynomial be added must be less than degree of the polynomial q(x).

78. Given that the sum of the zeroes of the polynomial $(a + 1)x^2 + (2a + 3)x + (3a + 4)$ is -1. Find the product of its zeroes.

79. If α and β are zeroes of the polynomials $f(x)=x^2-8x+k$ such that $\alpha^2+\beta^2=40$, find 'k'.

80. If α & β are zeroes of the polynomials $f(x)=x^2+px+45$ such that squared difference of the zeroes is 144 find the value of 'p'.

81. If α and β are zeroes of the polynomials $f(x) = kx^2 + 2x + 3k$ such that sum of zeroes is equal to the product of zeros, find the value of 'k'.

82. If α and β are zeroes of the polynomials $f(x) = 4x^2 - 8kx - 9$ such that zeroes are opposite in nature and equal in magnitude then, find the value of 'k'.

83. Find the zeroes of the polynomials $f(x) = x^3 - 12x^2 + 39x - 28$, if zeroes are $a - b, a, a + b$.

84. Find the value of 'a' and 'b' if polynomials $f(x) = x^3 - 3x^2 + x + 1$ has three zeroes as $a - b, a, a + b$.

85. If α, β and γ are the zeores of the polynomials $6x^3+3x^2-5x+1$, then find the value of $\alpha^{-1}+\beta^{-1}+\gamma^{-1}$

86. Find the values of 'a' and 'b' so that $x^4 + x^3 + 8x^2 + ax + b$ is divisible by $x^2 + 1$.

87. If polynomial $f(x) = x^4 - 6x^3 + 16x^2 - 25x + 10$ is divisible by another polynomial $g(x) = x^2 - 2x + k$, the remainder comes out to be $x + a$, find 'k' and 'a'.

88. If the polynomial $6x^4 + 8x^3 + 17x^2 + 21x + 7$ is divided by another polynomial $3x^2 + 4x + 1$, the remainder comes out to be $(ax + b)$, find a and b.

89. If $x^3 + 2x^2 + 4x + b$ is divided by $x + 1$, then the quotient and remainder are $x^2 + ax + 3$ and $2b - 3$ respectively. Find the values of a and b.

90. $4x^3 - 8x^2 + 8x + 1$, when divided by a polynomial g(x) gives $(2x - 1)$ as quotient and $x + 3$ as remainder. Find g(x).

91. What must be added to the polynomial $f(x) = x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is exactly divisible by $x^2 + 2x - 3$?

92. What must be subtracted from $8x^4 + 14x^3 - 2x^2 + 7x - 8$ so that the resulting polynomial is exactly divisible by $4x^2 + 3x - 2$.

93. Remainder on dividing $x^3 + 2x^2 + kx + 3$ by $(x - 3)$ is 21. Mody was asked to find the quotient. He was a little puzzled and was thinking how to proceed. His classmate Arvind helped him by suggesting that he should first find the value of k and then proceed further. Expalin how the question was solved? What value is indicated from this action of Arvind?

94. If $(x + a)$ is a factor of two polynomials $x^2 + px + q$ and $x^2 + mx + n$, then prove that $a = \frac{n - q}{m - p}$

95. If the sum of the zeroes of the polynomial $p(x) = (a+1)x^2 + (2a+3)x + (3a+4)$ is -1, then find the product of its zeroes.

ANSWERS

A. ZEROES OF POLYNOMIAL & VERIFY RELATIONSHIP BETWEEN ZEROES & COEFFICIENT

1. $-3, -2$ **2.** $5, 2$ **3.** $2, -3$ **4.** $\frac{3}{2}, \frac{-1}{3}$ **5.** $6, \frac{-1}{3}$ **6.** $\frac{1}{2}, \frac{1}{2}$

7. $-\sqrt{3}, \sqrt{3}$ **8.** $-\sqrt{5}, \sqrt{5}$ **9.** $\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ **10.** $0, -2$ **11.** $0, \frac{1}{3}$ **12.** $0, \frac{5}{12}$

13. $-\sqrt{3}, -\frac{7}{\sqrt{3}}$ **14.** $1, \sqrt{3}$ **15.** $\frac{\sqrt{3}}{4}, \frac{-2}{\sqrt{3}}$ **16.** $a, \frac{1}{a}$ **17.** $\frac{-b}{a}, \frac{c}{b}$ **18.** $1, \frac{1}{2}, -2$ **19.** $1, 1, 2$

B. FORMATION OF QUADRATIC POLYNOMIALS

22. $k[x^2 - 5x + 6]$ **23.** $k[9x^2 - 28x + 3]$ **24.** $k[x^2 + 11x + 30]$ **25.** $k[x^2 - (\sqrt{3} + \sqrt{5}) + \sqrt{15}]$

26. $k(x^2 - 7)$ **27.** $k[3x^2 - 2]$ **28.** $k[x^2 - 3\sqrt{3}x + 6]$ **29.** $k[4\sqrt{2} + (12 - \sqrt{6})x - 3\sqrt{3}]$

30. $k[x^2 - 6x + 4]$ **31.** $k[x^2 - 6x + 7]$ **32. a.** $k[x^2 + 5x + 6]$ **b.** $k[2x^2 - 5x + 2]$ **33.** $k[x^2 - 9]$

34. $k[3x^2 - 2x - 15]$

C. FORMATION OF CUBIC POLYNOMIALS

35. $k[2x^3 - 5x^2 - 4x + 3]$ **36.** $k[x^3 + 6x^2 + 11x + 6]$ **37.** $k[2x^3 + x^2 - 5x + 2]$

38. $k[x^3+17x^2+3x-140]$ **39.** $k[x^3-x^2-10x-8]$ **40.** $k[x^3-4x^2+x+6]$

41. **x** $k[x^3-19x+30]$ **42.** $k[x^3+17x^2+32x-140]$

D. DIVISION ALGORITHM & ITS APPLICATION

43. $2x-5, 0$ **44.** $x+4, 4$ **45.** $3x-1, 0$ **46.** $2x+3, 10$ **47.** Yes **48.** No

49. x^2-2x+3 **51.** $5, -4, -7$ **52.** $3x^2+5x-2, 0$ **54.** $8x^4+10x^3-5x^2+9x-8$

55. No, as degree of remainder is always less than the degree of divisor.

E. ZEROES OF A CUBIC OR BIQUADRATIC POLYNOMIAL GIVEN ITS ONE OR TWO ZEROES:

56. 2, –3 **57.** $-1, -10$ **58.** $3, -\frac{1}{3}$ **59.** $3, -\frac{1}{2}$ **60.** –5, 2 **61.** $\pm\frac{1}{\sqrt{2}}$

62. a. $1, \frac{1}{2}$ **b.** $\frac{1}{2}, 1$ **c.** 1, 2 **d.** 3, 4 **e.** $-2, 3$ **f.** $1, 2$ **g.** $1, 4$ **h.** $2, -3$ **i.** $7, -5$ **63.** $2, -3$

F. MISCELLANEOUS PROBLEMS (POLY.)

64. $k=7$ **65.** $k(x^2-24x+128)$ **66.** $a=-16$ **67.** 1 **68.** $\frac{1}{3}, \frac{7}{3}$ are roots $k=\frac{2}{3}$

69. $\frac{15}{7}$ **70.** $3x^2+10x-8$ **71.** $5x^2-12x+4$ **72.** $x^2-6x-72$

73. $7x^2-3x+1$ **74.** $8p^2-30p+25$ **75.** $21x^2-2x-8$ **76.** $\frac{-25}{12}$

77. $114x+77$ **78.** 2 **79.** 12 **80.** $(\alpha-\beta)^2=144$ **81.** $\frac{-2}{3}$

82. k = 0 **83.** 1, 4, 7 **84.** $1-\sqrt{2}, 1, 1+\sqrt{2}$ **85.** 5

86. a = 1, b = 7 **87.** k = 5, a = –5 **88.** a = 1, b = 2 **89.** a = 1, b = 0 **90.** $2x^2-3x+2$ **91.** x – 2

92. $14x-10$ **93.** $Q(x)=x^2+5x+6$, The action indicates helping nature of the student. **95.** 2