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DEEP PUBLIC SCHOOL

CLASS-XI

Subject - Mathematics

TIME: 3Hr

Term - I

M.M.100

Instructions

1. Questions 1 to 10 carry 1 mark each.
2. Questions 11 to 22 carry 4 marks each.
3. Questions 23 to 29 carry 6 marks each.

1. Let $f(x) = \begin{cases} x+3 & \text{if } x < 1 \\ 4x-2 & \text{if } 1 \leq x \leq 4 \\ x^2 + 5 & \text{if } x \geq 5 \end{cases}$

Find $f(-1)$, $f(3)$.

2. If $f(x) = x^2 - \frac{1}{x}$, then find the value of $f(x) + f\left(\frac{1}{x}\right)$.

3. Find the value of $\cos 55^\circ + \cos 125^\circ + \cos 300^\circ$.

4. If $\cot A = \tan(n-1)A$, then find A.

5. If $\sin A = \frac{1}{2}$, then what is $\sin 3A$?

6. Express $i^{17} + i^{18} + i^{19} + i^{20}$ in the form $a + ib$.

7. If ${}^nC_2 - {}^nC_1 = 35$, find the value of n.

8. If $f(x) = x^2$, find $\frac{f(1.2) - f(1)}{1.2 - 1}$.

9. If ${}^nC_2 - {}^nC_1 = 44$, then find n.

10. Express $(-\sqrt{3} + \sqrt{-2})(2\sqrt{3} - i)$ in the form $a + ib$.

11. Find the angle between the minute hand and the hour hand of a clock when the time is 5:20.

12. Prove that $2\sin^2 \frac{\pi}{6} + \operatorname{cosec}^2 \frac{27\pi}{6} \cos^2 \frac{2\pi}{3} = \frac{3}{2}$.

$\frac{60}{3} \times \frac{180}{\pi} \times \frac{30}{\pi}$

⑥ $\frac{35}{8} \times \frac{180}{\pi} \times \frac{260}{\pi}$

① $\frac{44}{70} \times \frac{180}{\pi} \times \frac{260}{\pi}$

$$\cos(A+B) - \cos(A-B) = \frac{2\sin(A+B)\sin A}{(A-B)} + 2\sin A \sin B$$

13. Prove that $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\frac{\sin 2x}{\cos 10x}$

14. How many 3-digit even numbers can be made using the digits 1, 2, 3, 4, 5, 6, if no digit is repeated?

15. If there are 15 points in which five points are collinear. How many straight lines can be drawn through these points.

16. Solve the equation $2x^2 + x + 1 = 0$.

~~BBB~~ ~~BBP~~
~~HHP~~ ~~BBP~~
RBP
HHB

17. Let $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$. Verify that

(i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

$$2 \sin A \sin B = \sin(A+B) + \sin(A-B)$$

(ii) $A \times C$ is a subset of $B \times D$

$$2 \cos A \cos B = \sin(A+B) - \sin(A-B)$$

18. Show that $\sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos 8\theta}}}} = 2\cos \theta$

$$2 \sin A \cos B$$

$$2 \cos A \sin B$$

19. Prove the following by using principle of mathematical induction for all $n \in N$:

$$1 + 3 + 3^2 + \dots + 3^{n-1} = \frac{(3^n - 1)}{2} \quad 1 + 3 + 3^2 + \dots + 3^{n-1} = \frac{(3^n - 1)}{2}$$

20. Solve the following system of inequalities graphically: $x + 2y \leq 10$, $x + y \geq 1$, $x - y \leq 0$, $x \geq 0$, $y \geq 0$.

21. Find a positive value of m for which the coefficient of x^2 in the expansion $(1+x)^m$ is 6.

22. Prove that $\cos^2 A + \cos^2 B - 2 \cos A \cos B \cos(A+B) = \sin^2(A+B)$.

~~Q5~~
~~Q2 + Q4~~

23. If a and b are distinct integers, prove that $a - b$ is a factor of $a^n - b^n$, whenever n is a positive integer.

24. Find the values of other five trigonometric functions if $\cot x = \frac{3}{4}$, x lies in third quadrant.

~~Q5~~
~~Q2 + Q4~~
~~Q3 + Q1~~

25. Find the domain of the function $f(x) = \sqrt{x^2 - 7x + 10}$

~~Q5~~
~~Q2 + Q4~~
~~Q3 + Q1~~
~~Q1 + Q3~~
~~Q2 + Q4~~
~~Q3 + Q1~~

26. How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

27. If the coefficients of T_r , T_{r+1} , T_{r+2} terms of $(1+x)^{14}$ are in arithmetic progression, then find the value of r :

~~2H~~
~~2x1@-1+1=0~~
~~2(1+1)=0~~
~~2/1125~~
~~1/1125~~
~~1/1125~~
~~1/1125~~
~~1/1125~~
~~1/1125~~

28. (i) If 8 parallel lines in a plane are intersected by a family of 10 parallel lines. Find the number of parallelograms formed.

(ii) Prove that $\tan 70^\circ = 2 \tan 50^\circ + \tan 20^\circ$

29. (i) How many numbers of 6 digits can be formed out of the digits of the numbers 567724? How many of the numbers so formed is even?

(ii) Prove the following by using the principle of mathematical induction for all $n \in N$:

$$1.3 + 2.3^2 + 3.3^3 + \dots + n.3^n = \frac{(2n-1)3^{n+1} + 3}{4}$$

$$\begin{array}{l}
 \begin{array}{c}
 x+2y \leq 10 \\
 2x+4y \leq 20 \\
 y \leq 5
 \end{array}
 \quad
 \begin{array}{c}
 x+y \leq 1 \\
 2x+2y \leq 2 \\
 y \leq 1
 \end{array}
 \quad
 \begin{array}{c}
 x-y \geq 0 \\
 2x-y \geq 0 \\
 y \geq x \\
 y \geq 1
 \end{array}
 \quad
 \begin{array}{c}
 x-3 \leq 0 \\
 2x-y \geq 0 \\
 y \leq 9 \\
 x-1 \leq 1
 \end{array}
 \end{array}$$