22 Sept.

ST. PAUL'S SCHOOL
FIRST TERMINAL EXAMINATION 2014-15
CLASS XI
MATHEMATICS

Time: 3 Hours **M.M : 100**

General Instructions:

(i) *All questions are compulsory.*

(ii) *The question paper consists of 29 questions divided into three sections, A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 7 questions of six marks each.*

(iii) *There is no overall choice.*

(iv) *Use of calculators is not permitted.*

SECTION A

Question numbers 1 to 10 carry 1 mark each.

Q.1 Let $f(x)=\begin{cases} x+3, x<1 \\ 4x-2, 1\le x\le 4 \\ x^2+5, x>4 \end{cases}$. Find $f(-1), f(3)$. (1)

Q.2 Find the domain of the function $f(x)=\dfrac{1}{\sqrt{x^2-1}}$. (1)

Q.3 Find the value of $\cos 55^\circ + \cos 125^\circ + \cos 300^\circ$. (1)

Q.4 Find the degree measure corresponding to the radian measure $\left(\dfrac{11}{16}\right)$. (1)

Q.5 If $2\cos\theta = x+\dfrac{1}{x}$, then find the value of: $2\cos 3\theta$? (1)

Q.6 Express $i^{17}+i^{18}+i^{19}+i^{20}$ in the form of $a+ib$. (1)

Q.7 Find 'n', if ${}^{25}C_{n+5}={}^{25}C_{2n-1}$. (1)

Q.8 How many elements has P(A), if $A=\phi$? (1)

Q.9 Find the conjugate of $\frac{3-4i}{1-i}$. (1)

Q.10 Solve: $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$. (1)

(1 * 10 = 10)

SECTION B

Question numbers 11 to 22 carries 4 marks each.

Q.11 How many different words can be formed of the letters of the word 'MALENKOV' so that:

(i) First letter is a vowel.

(ii) No 2 vowels are together.

(iii) Vowels may occupy odd places.

(iv) Vowels being always together.

Q.12 Let A, B and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$. Show that $B = C$.

OR

If $A = \{3,5,7,9,11\}, B = \{7,9,11,13\}, C = \{11,13,15\}$ and $D = \{15,17\}$; find

$(i) A \cap (B \cup C)$

$(ii) (A \cap B) \cap (B \cup C)$

$(iii) (A \cup D) \cap (B \cup C)$

Q.13 Prove by using the principle of mathematical induction for all $n \in N$:

$$\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}.$$

Q.14 Show that the middle term in the expansion of $(1+x)^{2n}$ is $\frac{1.3.5......(2n-1)}{n!} 2^n x^n$ where n is a positive integer.

OR

Find the term independent of x in the expansion of $\left(\sqrt[3]{x} + \frac{1}{2\sqrt[3]{x}}\right)^{18}, x > 0$.

Q.15 Three vertices of a parallelogram $ABCD$ are $A(3, -1, 2)$, $B\ (1, 2, -4)$ and $C\ (-1, 1, 2)$. Find the coordinates of the fourth vertex.

Q.16 A group consists of 4 girls and 7 boys. In how many ways a discipline team of 6 members be selected if team has 2 girls. If you are asked that how many girls and boys should be taken in the discipline team, then what will be your views?

Q.17 Solve: $\frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{\tan 8\theta}{\tan 2\theta}$.

OR

Prove that: $\cos 2\theta \cos\frac{\theta}{2} - \cos 3\theta \cos\frac{9\theta}{2} = \sin 5\theta \sin\frac{5\theta}{2}$

Q.18 If α and β are complex numbers with $\beta = 1$, then find $\left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right|$.

OR

Convert the complex number $z = \frac{i - 1}{\cos\pi/3 + i\sin\pi/3}$ in the polar form.

Q.19 Solve the following equation: $\sqrt{3}\cos x - \sin x = 1$.

Q.20 (i) Let $f, g : R \to R$ defined, respectively by $f(x) = 3x + 2, g(x) = 5x - 7$. Find $f + g, f - g$ and $\frac{f}{g}$.

(ii) Let $A = \{a, b\}$ and $B = \{c, d, e\}$. Find the number of relations from A to B.

Q.21 In how many ways can the letters of the word BANANA be arranged so that the two N's do not appear adjacently?

Q.22 Find the domain and range of the function: $f(x) = \frac{x}{1 + x^2}; x \in R$.

(12 * 4 = 48)

SECTION C

Question numbers 23 to 29 carries 6 marks each.

Q.23 Prove that:

(i) $\tan 7\theta - \tan 5\theta - \tan 2\theta = \tan 7\theta \tan 5\theta \tan 2\theta$.

(ii) Prove that: $\cos\alpha + \cos\left(\alpha + \frac{2\pi}{3}\right) + \cos\left(\alpha + \frac{4\pi}{3}\right) = 0.$

OR

(i) Prove that: $\sin^2\left(\frac{\pi}{8} + \frac{A}{2}\right) - \sin^2\left(\frac{\pi}{8} - \frac{A}{2}\right) = \frac{1}{\sqrt{2}}\sin A.$

(ii) $\sin x + \sin 3x + \sin 5x + \sin 7x = 4\cos x \cos 2x \sin 4x$.

Q.24 In a survey it was found that 21 people liked product A, 26 liked product B and 29 liked C. If 14 people liked products A and B, 12 people liked products C and A, 14 people liked products B and C and 8 liked all the three products. Find how many liked :

(i) product C only;

(ii) products A and C but not product B;

(iii) at least one of the three products.

Q.25 Solve the following system of inequalities graphically:

$$x + 2y \le 10,$$
$$x + y \ge 1,$$
$$x - y \le 0$$
$$x \ge 0,$$
$$y \ge 0$$

Q.26 Prove that:

$$\cos A \cos 2A \cos 4A \cos 8A = \frac{\sin 16A}{16 \sin A}$$

OR

Prove that: $\tan 20^\circ \tan 40^\circ \tan 80^\circ = \sqrt{3}.$

Q.27 Find 'n', if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6}:1$.

Q.28 (i) How many numbers greater than 1000000, can be formed by using digits 1, 2, 0, 2, 4, 2,4 ?

(ii) In how many ways can 5 girls and 3 boys be seated in a row so that no two boys are together?

Q.29 (i) If $|z|=1$, then prove that $\frac{z-1}{z+1}, (z \neq -1)$ is a purely imaginary number.

(ii) If $|z+i|=|z-i|$, find z.

(7 * 6 = 42)